Use the method of separation of variables to show that the 1-D heat conduction problem: find $u = u(x,t)$ so that

$$\begin{aligned} \frac{\partial u}{\partial t}(x,t) - \alpha \frac{\partial^2 u}{\partial x^2}(x,t) &= 0,\ 0 < x < l,\ t \geq 0 \\ u(0,t) &= 0,\ t \geq 0 \\ u(l,t) &= 0,\ t \geq 0 \\ u(x,0) &= f(x),\ 0 \leq x \leq l \end{aligned}$$

has the solution

$$u(x,t) = \sum_{n=1}^{\infty} c_n \sin \frac{n\pi x}{l} \exp\left[-\alpha \left(\frac{n\pi}{l}\right)^2 t\right]$$

where

$$c_n = \frac{2}{l} \int_0^l f(x) \sin \frac{n\pi x}{l} dx.$$