Abstracts

MAA-SE/SIAM-SEAS Meeting

Clemson University, March 21-22, 2003

FRIDAY MARCH 21st

1:00-2:10 General Session I

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Optimization is Everywhere

Twenty years ago, "optimization" was a relatively precise technical term in computer science, mathematics, and operations research; today, it is routinely attached to essentially every aspect of life, including political campaigns, hair care products, and dental hygiene. But there are still people, of whom the speaker is one, who view optimization as an array of mathematical and computational techniques for identifying and finding the best value of an objective function subject to constraints.

The field of optimization in this classical sense has, like its name, undergone sweeping changes during the past twenty years. One of the most visible, called "the interior-point revolution", began in 1984 with Karmarkar's announcement of a fast polynomial-time algorithm for linear programming. Since then, interior-point methods have turned out to be a fascinating mixture of old and new ideas, with applications ranging from linear programming to approximation algorithms for NP-hard problems. This talk will describe the evolution of modern interior methods from the controversies of 1984 to the present, and will try to convey, emphasizing both research and pedagogy, the excitement that remains after almost 20 post-revolution years.

2:20-4:20 Concurrent Sessions

Number Theory

1. *Determining Mills' Constant and a note on Honaker's Problem*, Chris K. Caldwell and Yuanyou Cheng, University of Tennessee at Martin

In 1947 Mills stated that there exists a constant A such that $\lfloor A^{3^n} \rfloor$ is a prime for every positive integer n. Determining A requires determining an effective Hoheisel type result on the primes in short intervals. Under the Riemann Hypothesis, we show that there exists at least one prime between every pair of consecutive cubes and determine that the least possible value of Mills' constant A begins with~1.3063778838 (we determine this value to over 2280 decimal places). We also apply the Cramer Conjecture to Honaker's problem in a related context.

2. *A Connection Between Ordinary Partitions, Rogers-Ramanujan Partitions, and 2-Color Frobenius Partitions*, Louis W. Kolitsch, The University of Tennessee at Martin

In this paper a connection between three types of partitions is established. The connections are verified algebraically as well as through direct transformations.

3. *What Is Special About The Kaprekar Routine*? Kevin Peterson, Lynchburg College and Hari Pulapaka, Stetson University

The Kaprekar Routine is a famous mathematical procedure involving the digits of a positive integer. This talk attempts to revive the magic and lure of this fascinating procedure by exploring the problem of classifying all the "Kaprekar-type" routines for 3-digit, r-adic numbers. Additionally, some open problems that are suitable for undergraduate student exploration are presented.

4. *Structural Properties of c(Z_{pq})-Sets*, Michael Freeze, University of North Carolina at Wilmington

Let G be a finite abelian group. The critical number c(G) is the smallest positive integer n such that every element of G occurs as a sum of distinct elements over any subset A of G with cardinality n. We investigate the distributional structure of sets A achieving c(G) where G is a cyclic group of order pq for odd primes p and q.

5. *The Bracelet Problem -- Fibonacci Numbers mod m*, David R. Stone, Georgia Southern University

In the Number Bracelets game, students string numbered beads, where each bead has a digit on it. After an initial choice of two beads, each subsequent bead is chosen by the Fibonacci recurrence relation mod 10: d[n+1] = d[n] + d[n-1] mod 10. Eventually the sequence will loop back upon itself, closing the bracelet. Questions abound: which sequences appear, how many, what are their lengths? -- especially when the game is played with other moduli and other second-order recurrence relations. These questions can be treated and investigated at several different mathematical levels, from early

elementary school upwards.

6. *On 1 (mod 3) Prime Numbers*, Shan Manickam, Western Carolina University and Swarnameenakshi Manickam, Yale University

A conjecture of Gilman (1997) states that each 1 (mod 3) prime has the representation m2 + mn+n2, where m, n are unique positive integers. An equivalent form of this conjecture, that each 1 (mod 3) prime number is of the form x2 + 3 y2 where x, y are integers, is proved and a computational framework to determine m, n, x and y is shown.

Algebra & Discrete Math

1. *Parsing Permutations*, Jeff Clark, Elon University

This presentation will explore the details of desiging a computer program that can read a permutation as a string in cycle notation.

Along the way I will describe how to use a deterministic finite automaton as a model for such an algorithm as well as its limitations.

2. *What the heck are Rado numbers?* Carl Mueller, Georgia Southwestern State University

In this talk I will discuss the basics of an area of combinatorics known as Ramsey theory. The topic under investigation deals with colorings of the natural numbers and solutions to equations involving integers that are all colored the same. After a brief introduction to this area, a specific problem that was recently solved by the presenter will be discussed.

3. *On the Capability of a Metacylic Group*, Jim Beuerle, Elon University

The Nonabelian Tensor Square (NATS) of a group G is generated by the symbols $g \otimes h$ and defined by the relations $gg' \otimes h = ({}^{g}g' \otimes {}^{g}h)(g \otimes h)$, $g \otimes hh' = (g \otimes h)({}^{h}g \otimes {}^{h}h')$, where $g,g',h,h' \in G$ and the action is defined by ${}^{x}y = xyx^{-1}$. This talk will introduce techniques that are used to determine NATS. In particular, the talk will concentrate on metacyclic groups and how their NATS can be used to determine if the group is capable. A capable group is one that is isomorphic to an inner automorphism group.

4. *A Trick for Introducing Algebraic Coding Theory*, Jeffrey Ehme and Colm Mulcahy, Spelman College

In this presentation we will present an application from elementary algebraic coding theory to card magic. Our mathemagician will

illustrate the trick and explain the mathematics involved. This trick is useful for illustrating some key ideas in coding theory.

5. *Grünbaum Colorings of Triangulations of the Sphere*, Eric Gottlieb and Kennan Shelton, Rhodes College

A Grünbaum coloring of a triangulation is an assignment of colors to edges so that the edges about each face are assigned unique colors. In this paper we examine the color induced subgraphs given by a Grünbaum coloring of a triangulation and show that the existence of connected color induced subgraphs is equivalent to the Three Color Theorem.

6. *Counting on Hypercubes,* Stephen Davis, Davidson College

Warm-up: take a cube and find a labeling of its vertices with 1, 2, ..., 8, so that the six "face-sums" are all the same. This warm-up problem has a solution (actually, several different solutions). What is an appropriate generalization of this problem to a 4-dimensional hypercube? How about an n-dimensional hypercube (n = 2, 3, 4, ...)? This talk presents one solution to this last question. The presentation is intended to be student-friendly; undergraduates are invited to attend.

Matrix Theory & Numerical Linear Algebra

1. *Spectra of Leslie Adjacency Matrices with Applications*, Bruce W. Atkinson, Samford University

The Leslie model of population growth starts by dividing a population into n consecutive age groups. The (i,j) entry of the

corresponding Leslie matrix is the percent of the population from group j that is represented in group i in the next generation via births or survival. Typically the first row consists of leading zeros followed by non-zero entries; these represent birth rates. For i > 1 there is a single non-zero entry at position i-1 ; these are the survival rates. A Leslie Adjacency matrix is a Leslie matrix where each non-zero entry is a one, and where the first row has leading zeros and at least two non-zero entries. It is shown that every Leslie Adjacency matrix is primitive. A formula for the characteristic polynomial is derived and used to estimate the spectral

radius. Perron-Frobenius theory then provides some asymptotic results concerning the powers of the matrix. Since the 2-by-2 Leslie Adjacency matrix generates the standard Fibonacci sequence, then the general results for the n-by-n matrices have asymptotic consequences for certain generalizations of Fibonacci sequences. These results also apply to the digraph corresponding to the Leslie Adjacency matrix. This, in turn, has applications for the population model indicated above.

2. *Subproper and regular splittings for a restricted rectangular system*, Xiezhang Li and Yimin Wei, Georgia Southern University

Let Ax = b be a rectricted rectangular and consistent linear system, where A is an m by n matrix and x is in a subspace T of C^n.
The concept of subproper splitting A=M-N introduced by Neumann is generalized. A necessary and sufficient condition on the
subproper splitting such that the iterative sequence converges to a solution of Ax = b is given. Monotonicity and the concept of regular subproper splitting are used to study convergence. Numerical examples are given to verify our conclusions.

The Superiority of a New Type (2,2)-Step Iterative Method over the Related Chebyshev Method

Mei-Qin Chen[1] and Xiezhang Li[2]

Abstract

A (2,2)-step iterative method for solving $\mathbf{x} = \mathbf{T}\mathbf{x} + \mathbf{c}$ generates a sequence of estimates $\{\mathbf{x}_k\}$ as follows:

$$\mathbf{x}_0{=}\mathbf{c},\ \mathbf{x}_1{=}\ \mathbf{T}\left(\frac{v_0}{u_0}\mathbf{x}_0\right){+}\mathbf{c},$$
$$\mathbf{x}_k{=}\frac{1}{u_0}\left(\mathbf{T}\left(v_0\mathbf{x}_{k-1}{+}v_1\mathbf{x}_{k-2}\right){+}\mathbf{c}-\left(u_1\mathbf{x}_{k-1}{+}u_2\mathbf{x}_{k-2}\right)\right),\ \text{for } k\geq 2,$$

where $u_0+u_1+u_2 = v_0+v_1 = 1$ and $u_i's$ and $v_i's$ are given by

$$\psi(w) = w\frac{u_0+u_1w^{-1}+u_2w^{-2}}{v_0+v_1w^{-1}} \quad \text{for } |w|>\rho>0.$$

When the spectrum of $\mathbf{T}$ is contained in the closed ellipse

$$\Omega = \left\{(x,y)\ |\ \frac{(x-d)^2}{a^2(1+b)^2}+\frac{y^2}{a^2(1-b)^2}\leq 1\right\}$$

where $a>0$, $0<b<1$ and $a(1+b)+d<1$, it is known that the 2-step method ($v_1=0$) generated by ($\psi=\Psi$)

$$\Psi(w) := a\left(w+\frac{b}{w}\right)+d,\ \text{for } |w|>1$$

achieves the same asymptotic rate of convergence (ARC) as the optimal Chebyshev method determined by $\partial\Omega$. In the case where $b<\sqrt{5}-2$ and there is no eigenvalue of $\mathbf{T}$ in the first quadrant of $\partial\Omega$, the (2,2)-step method generated by

$$\Psi_c(w) := a\left(w+\frac{b}{w-c}\right)+d-\frac{abc}{1-c},\quad \text{for } |w|>1$$

improves ARC of the optimal Chebyshev method.

In this talk, we first present a new function Ψ_{cf},

$$\Psi_{cf}(w) := a(w+\frac{f}{w-c})+d+ab-\frac{af}{1-c},\quad \text{for } |w|>1$$

and its properties. We then show that the (2,2)-step method generated by Ψ_{cf} improves ARC of the optimal Chebyshev method in the case where $b\geq\sqrt{5}-2$. Numerical examples are given to illustrate the superiority of the new type (2,2)-step method over the optimal Chebyshev method and comparisons with other iterative methods are also reported.

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4. *Inertia Sets of Symmetric Sign Pattern Matrices*, Frank J. Hall and Zhongshan Li, Georgia State University

A sign pattern matrix is a matrix whose entries are from the set {+, -, 0}. The symmetric sign pattern matrices that require unique inertia have recently been characterized. In this talk the inertia sets of symmetric sign pattern matrices are more generally investigated. In particular, nonnegative tri-diagonal sign patterns and the square sign pattern with all + entries are examined. The inertia set of the square pattern with all + off-diagonal entries and zero diagonal entries is then analyzed. The types of inertias which can be in the inertia set of any sign pattern are also obtained. Specifically, certain compatibility and consecutiveness properties are established

5. *The Recursive Inverse Eigenvalue problem*, Marina Arav, Georgia State University

The paper is on the recursive inverse eigenvalue problem for matrices, in which for each leading principle submatrix an eigenvalue and associated left and right eigenvectors are assigned. The existence and uniqueness results and some explicit formulas are proven. Applications to nonnegative matrices, Z-matrices, M-matrices, Stieltjes matrices and inverse M-matrices are considered.

The Teaching of Statistics

1. *The Availability Misconception in Probability and Statistics: An Investigation of High School Students*, Rhonda C. Porter, Florida A & M University

The purposes of this study were to determine if the Availability misconceptions in stochastics is present in high school students; to
determine if demographics (i.e., ethnic background, gender, grade, and family socioeconomic level) are related to Availability; and to determine if academic factors (academic track, on or off mathematics grade level, and taking honors classes) are related to Availability. Two hundred high school students were asked three purposeful questions addressing the Availability misconception. The results showed that a majority of the high school students in this study have answered at least one of the purposeful questions using the Availability misconception. The results also showed that gender and students who were enrolled in honors mathematics classes showed a positive significant relationship to the misconception of Availability.

2. *Teaching Statistics: When is the Sample Size Large Enough?* Richard Stephens, University of Alaska Southeast

In many Mathematics Departments mathematicians are called upon to teach Elementary Statistics, but do we investigate the mathematical weaknesses of the standard statistical practices presented in the textbooks we use for such courses? Although there
are many of these weaknesses, this presentation will only address issues related to the sample size needed for estimating and testing hypotheses concerning the value of a population proportion. The standard "rules of thumb" will be analyzed and alternatives
will be proposed.

3. *FreeCell, Common Sense and Statistics*, Paul Baker, Catawba College

While playing the Windows-ubiquitous computer game FreeCell, some fairly sophisticated statistical questions arise naturally. Applying some common sense to the questions can give students an intuitive feel for probability, random distribution, expected values and hypothesis testing. Once motivated by FreeCell, students can be guided to use the rules of statistics to verify or discount the common sense solutions.

4. *Incorporating Activities and Web-Based Materials into Post-Calculus Probability and Statistics, A Preliminary Report*, Dr. Tracy Goodson-Espy, University of Alabama in Huntsville, Dr. M. Leigh Lunsford, Athens State University, Dr. Ginger Holmes Rowell, Middle Tennessee State University

As part of a collaborative NSF Adaptation and Implementation (A&I) grant, the authors are incorporating activities and web-based materials into post-calculus probability and statistics classes at ASU and MTSU. The materials adapted and implemented are from the NSF projects--*Virtual Laboratories in Probability and Statistics (VLPS)*, and *A Data-Oriented, Active Learning, Post-Calculus Introduction to Statistical Concepts, Methods, and Theory (SCMT)*. In addition to adapting and implementing these materials, this project will focus on evaluating their effectiveness in three distinct types of probability and statistics classes and the processes used to integrate the materials into these contexts. Project dissemination will include a workshop for faculty in the summer of 2004. This presentation will describe the new methods and materials, the motivation for using these methods and materials, general impressions of the A&I process to date, lessons learned, project assessment, and preliminary assessment results. Please visit the project website to learn more. http://www.athens.edu/NSF_Prob_Stat/

5. *Teaching an Introductory Statistics Course on the Internet*, Lothar A. Dohse, University of North Carolina at Asheville

The University of North Carolina at Asheville now offers a distance learning Introductory Statistics Course. The basis of this course is to have students submit work frequently and get immediate feedback from the instructor. A discussion will be given as to what the perceived new challenges are for the students, as well as what hurdles the instructor must

overcome in order to give such a course effectively. The presentation will focus on the preliminary results, and highlight some of the author's early misconceptions, failures and successes.

6. *If technology has revolutionized the teaching of Statistics, Why are we still teaching essentially the same course?* Patricia Humphrey, Georgia Southern University

Guidelines from the Committee on Undergraduate Programs in Mathematics (CUPM) in 1992 stated "Every Mathematics major needs a course focussed on analyzing data." It further states all college graduates should possess quantitative reasoning skills "to use mathematical methods to solve real-world problems." In many cases, an introductory Statistics course meets these needs. Technology in the form of computer packages such as Minitab, JMP, DataDesk; Web applets, and hand-held technology such as TI graphing calculators has drastically changed the way in which computations for these courses can be and are done. Presented are some examples of old and curent texts and supplements which prompt the question posed in the title of the talk. Some potential options and answers are discussed.

Undergraduate Student Papers I

1. *Generalizations and Analogues of the Pythagorean Theorem*, Jessica Munley, Elon University

One of the most well-known theorems in mathematics is the Pythagorean theorem. This theorem is studied in every geometry class, and dates back thousands of years. So after all this time, how can we still learn new things from the Pythagorean theorem? There are many generalizations and extensions of this theorem. For example, by placing arbitrary parallelograms on two sides of any triangle, it can be found that the sum of the areas of these two parallelograms is equal to the area of another parallelogram constructed on the third side of the triangle if the height it chosen appropriately. Can other shapes be used in place of parallelograms? This presentation will also explore three dimensional generalizations using boxes, tetrahedrons, projections onto the three coordinate planes.

2. *An Investigation of Excevians and Extriangles*, J Brian Parker, Elon University

Suppose you create squares on the edges of a triangle and connect adjacent corners of the squares to form triangles, called extriangles. This presentation will investigate the properties of the cevians of extriangles, called excevians. Specifically, the special cevians of these extriangles are called exomedians, exaltitudes, exangle bisectors, and experpendicular bisectors respectively. It turns out that each of these excevians have interesting points of concurrency. If rectangles with equal proportion of length to width are used in place of squares, will similar properties hold? We will further investigate properties of excevians by altering our definition of extriangles by unfolding the edges of a triangle by any equal measure to see what sort of properties will then exist. The research concludes by drawing the conclusion that extriangles formed by constructing any polygon with equal base angles on the sides of an original triangle will share the property that the exaltitudes and exomedians will be concurrent along the Kiepert hyperbola.

3. *Binomial Coefficients, Trinomial Coefficients and the Pascal Triangle*, Jeanette Olli, Elon University

The Pascal triangle is full of symmetry and patterns and it seems as if more are being discovered ever day. The values in the triangle can be thought of as the coefficients in the expansion of $(a+b)^n$ and it is from thinking of them in this way that more can be discovered. The value of n can be both positive and negative, and looking at at the cases creates a Pascal hexagon. Thinking in three dimensions, we start with a tetrahedron, and algebraically we can find the complete expansion of any m-nomial.

4. *Vertex Magic*, Katherine Cunningham, Elon University

One can find very intrigueing properties when applying vertex-magic labelings to cycle graphs. Bounds are found for the magic number for any given cycle graph. The vertex-magic labeling of a cycle graph with an odd number of vertices is very different from that of a cycle graph with an even number of vertices. Algorithms are shown for producing vertex-magic graphs where the magic number is the minimum or maximum possible value. There are additional algorithms for finding vertex-magic labelings for cycle graphs with an odd number of vertices.

5. *Factoring Large Permutation Groups*, Kathleen Iwancio, Elon University

A permutation is a one-to-one and onto mapping that rearranges points in a set $P = \{p_1, p_2, ..., p_n\}$. A permutation group is a set of permutations that form a group under composition. Some permutation groups are very large, making them more difficult to work with than a small group where we can list out all the elements easily. Working with large permutation groups is simplified with some knowledge of orbits, coset representatives, and Schreier generators. Schreier generators are used to generate the stabilizer of a particular point. By stabilizing different points, the permutation group can be factored, which simplifies finding elements in the group and counting them. Two important examples of large permutation groups are the Rubik's cube group and the Monster group. The Monster group is the largest group known of and may play an important role in string theory.

6. *Random Growth of Cell Blocks*, Joseph A. Johnson, East Tennessee State University

Consider (in two or more dimensions), cells that might be square or triangular or hexagonal. These are added to each other sequentially forming an ordered cell block (or what physicists call an "animal"). We examine small cases and the

asymptotics of the number of animals as n grows large. Parametric considerations play a key role in the development.

Undergraduate Student Papers II

1. *Modeling the Laundry Problem using Circle Maps*, Stuart Bateman, University of North Carolina at Asheville

Imagine that a college student throws all of her dirty clothes into a single basket. Suppose that the same portion of a full load of laundry goes into the basket every day. To save money, she does not wash her clothes until there is at least a full load of laundry in the basket. Further, the student keeps a journal of when she does laundry, writing a one on days she does laundry and a one on days she does. The sequence of zeros and ones in her journal is called the laundry itinerary. We use a circle map with rigid rotation to model when the student does laundry. Using rotation numbers we analyze the dynamics of the map and its bifurcations. We also demonstrate criteria to determine if a given sequence is a laundry itinerary. To aid in numerical study we also generate an algorithm for computing laundry itineraries. Laundry itineraries are related to Sturmian sequences and Fibonacci words. As with these other two models, the laundry problem has applications in complexity theory and neural networks.

2. *Paths That Turn at a Constant Rate: Special Curves in the Hyperbolic Plane*, Rob McLean, Davidson College

In the Euclidean plane there are only two curves of constant curvature, the line and the circle. However, in the hyperbolic plane the curves of constant curvature are more abundant. I will summarize two arguments that classify the curves of constant curvature in the hyperbolic plane. One argument is strictly model dependent and relies on the differential geometry of the Poincare upper half plane and the Poincare disk. The other argument is more algebraic and focuses on the orbits of points under subgroups of hyperbolic motions.

3. *Strategies for re-establishment of the American chestnut in the Appalachians*, Amelia Nutter, University of North Carolina at Asheville

In hopes of reintroducing the historically invaluable but blight ridden American chestnut tree (Castanea dentata) to the Appalachians, breeding programs have been developed to produce a blight-resistant hybrid of American chestnut and Chinese chestnut (Castanea mollissima). Once the hybrid has been established, the manner in which these blight-resistant trees should be reintroduced on forest sites is still in question. This study attempts to determine the best circumstances under which to distribute the seedlings. The experiment was conducted in Bent Creek Experimental Forest in western North Carolina over a five-year period. One hundred and twenty seedlings were planted in two blocks of three canopy levels each. Four treatment combinations of grow tube and fertilizer were implemented within each canopy level. To assess effects on survival, a statistical split-plot design was employed with three canopy treatments as the whole plot factor and four combinations of fertilizer and grow tube as the split-plot factors. Grow tubes were discovered to have a significant positive effect on survival while the application of fertilizer produced a significant negative effect on survival. Canopy type did not significantly effect seedling survival.

4. *Assessment of Lead Levels in Dust, Soil and Paint in Durham, North Carolina*, Alyssa Dillow, The University of North Carolina at Asheville

It is well known that high lead levels in children can have serious physical and mental effects, such as learning disabilities and decreased growth. For this reason the Partnership for Advancement of Children's Health (PEACH) conducted a study of approximately 650 homes, apartments and daycares, focusing on those built before 1978, which were sampled for lead levels in dust, before and after a lead wash, soil and paint. The data collected from this study was analyzed to test the effects of washing, age of home, room and location within the room on the amount of lead in dust and paint. In addition, the amount of lead in soil was examined to determine the correlation between lead levels, age of home and location. After a split-plot statistical analysis was performed, it was determined that the year the house was built and the room the sample was taken from were significantly related to the quantity of lead in dust. Paint and soil varied from dust in that the year the house was built and location were significant factors in determining lead levels in these substances. Evidence shows a statistical difference in lead levels between pre-wash and post-wash dust samples. Despite the difference, the lead wash does not eliminate the high lead level problem in pre-1978 structures.

5. *An Investigation of the Ordered "Look-and-Say" Sequence*, Jason Grigsby, Birmingham-Southern College.

We study a variation of the "look-and-say" sequence due to John H. Conway and Richard Guy. A graphical user interface was developed to investigate this new sequence. Results obtained from the Internet-based tool are provided.

6. *Geometry and Monte Carlo Simulation in Election Modeling*, Emily Marcato, Samford University

The election modeling of this talk utilizes several areas of mathematics - linear algebra, geometry, probability, discrete mathematics, and graph theory. A voter is considered a ranking among n candidates. An election mapping is a function determining election outcomes based on collections of individual voter rankings. Unfortunately, some mappings make paradoxes possible; e.g., the plurality winner might not be the winner in pair-wise comparisons. The paradox of cycles can occur where a cycle among three candidates consists of A beating B, B beating C, and yet C beating A.The election profile consists of proportions of voters with the same rankings. For pair-wise comparisons a linear transformation

maps the profile into a representation cube. Gaussian elimination determines exactly which profiles result in cycles. In a random election the probability of cycles is calculated through Monte Carlo simulation. Digraphs display election results. More digraphs contain cycles than not among four candidates. This suggests the probability of cycles is high for a large number of candidates, which will be shown. Mathematica is used to perform simulations. The study concludes by determining conditions under which cycles are unlikely; e.g. making part of the profile deterministic. Paradoxes in committee elections are also analyzed.

Graduate Student Contributed Papers I

Numerical Method for Sand Pile Formation (Numerical PDEs, Solid Mechanics)

Christopher Kuster (North Carolina State University)
Dr. Pierre Gremaud (North Carolina State University)
Robert Buckingham (Duke University)

We consider the surface profile of grain flowing into a hopper from a point source. This surface is constrained to have constant gradient due to the balancing of gravitational forces with the internal friction of the grain. In the case where the domain is unobstructed, the solution is that of a conical pyramid with slope of δ. When the region contains obstacles, the surface profile is more complicated. This situation can be represented by the Eikonal equation, $\|\nabla\phi\| = R$, where $R = \tan\delta$ in unobstructed regions, and $R = \infty$ inside obstacles. This problem is related mathematically to travel time problems.

A standard method of solving such problems is the fast marching method. We present here, an algorithm that keeps the marching characteristics of fast marching while maintaining accuracy in regions containing obstacles unlike the standard method. This method is useful in cases where the geometry of the region is known. We present results pertaining to the surface profile of obstructed sand piles.

2. *Cone-Based Modeling of Preferences in Multicriteria Optimization*, Brian J. Hunt, Clemson University

The selection of a preferred decision in multicriteria optimization requires preferences from the decision maker. A convex, polyhedral cone, represented by a homogeneous system of linear inequalities with coefficient matrix *A*, may be used to model these preferences. When *A* is the identity matrix *I*, the case is the well-known Pareto preference. We analyze the more general case when *A* is a specially structured matrix with nonnegative entries and examine how matrices of this type model preferences and affect the set of nondominated solutions.

3. *The Fractional Advection Dispersion Equation*, John Paul Roop, Clemson University

In this talk we shall discuss a generalization of the traditional advection-dispersion equation to include fractional order dispersive terms. In order for this, we introduce the notion of a fractional order differential operator and derive the modeling equations using both stochastic and conservational principles, noting that each case relies on a strong non-local interaction of the elementary units--a property of anomalous diffusion. We present a least squares finite element
formulation of the steady-state problem, existence and uniqueness results, error estimates, and numerical results in one and two dimensions. Some calculations for the transient case are also included.

4. *Numerical Simulation of Diffusion of Second Messengers in Visual Transduction*, Harihar Khanal, University of Tennessee

A crucial step in the process of phototransduction, whereby light is converted into an electrical response in retinal rod and cone photoreceptors, involves interaction and diffusion of cytoplasmic signaling molecules, termed second messengers, in the cytosol. A computational model for the interaction and diffusion of the second messengers (cyclic Guanosine Monophosphate and Calcium) during the activation phase of phototransduction in retinal rod cells is described and results of numerical simulations are presented.

5. *Using Quantitative Methods to Improve Your Tennis*, Chris Valis, Wake Forest University

In singles tennis, two players serve in alternate games. For each point, the server uses at most two serves. This paper describes the quantitative methods applied in developing a computer program which, for each player: (1) uses dynamic programming to select the best first and second serve and (2) uses absorbing Markov chain methods in computing probabilities of winning a point, game, set and match. Results show how to improve one's serve in order to maximize their chances of winning.

Special Session on Discrete Mathematics

Organizers: Robert E. Jamison and Renu Laskar, Clemson University

This session is intended to provide an informal opportunity for researchers in Discrete Mathematics in the SouthEast to share their current research interests. We wish especially to bring together not only researchers from research universities but also faculty with research interests in Discrete Mathematics from 4-year colleges.

Special Session on Discrete Mathematics I

FRIDAY, March 21, 2:20 - 2:45

Title: Generalizing Pancyclic and k-Ordered Graphs

Ronald J. Gould, rg@mathcs.emory.edu, Emory University
Co-authors: R. Faudree, M. Jacobson, L. Lesniak

Given positive integers $k \leq m \leq n$, a graph G of order n is (k, m)-*pancyclic* if for any set of k vertices of G and any integer r with $m \leq r \leq n$, there is a cycle of length r containing the k vertices. Minimum degree conditions and minimum sum of degree conditions of nonadjacent vertices that imply a graph is (k, m)-pancylic are proved. If the additional property that the k vertices must appear on the cycle in a specified order is required, then the graph is said to be (k, m)-*pancyclic ordered*. Minimum sum of degree conditions for nonadjacent vertices that imply a graph is (k, m)-pancylic ordered is also proved. Examples showing that these constraints are best possible are provided.

FRIDAY, March 21 2:50 - 3:15

Cylindrical Braids

Dave Peifer, dpeifer@unca.edu, UNCAsheville

A standard braid on n strings is constructed by attaching n strings to a straight bar,

Say that P has the *splitting property* if every maximal antichain of P splits. Ahlswede, Erdős and Graham introduced these notions and proved that every finite Boolean lattice has the splitting property. Duffus and Sands used the splitting property to study maximal antichains in distributive lattices, characterized those distributive lattices with the splitting property, and introduced the idea of a splitting number for any finite ordered set and any class of finite ordered sets.

In studying the splitting numbers of products of two chains, or *grids*, some surprising and still mysterious links to Pell numbers and a "Pascal-like" triangle arise.

This is joint work with Bill Sands [The University of Calgary].

FRIDAY, March 21 3:50 - 4:15

Monster in a Box: The Interplay of Integer Sequences

Evan B. Wantland, wantland@warren-wilson.edu, Warren Wilson College

Here's the situation:

A)You have a set of labeled tennis balls

B)Conveniently, sitting beside you is a box with a tennis-ball-hating monster in it. (Its a big box, ok?)

C)You and the monster begin to play a 'game' where each step in the game consists of two actions:

1)You throw the next two labeled tennis balls into the box.

2)The monster throws any tennis ball in the box back out of the box (and far away from you, his tormenter).

Here's a question:

After n steps, how many different combinations of labeled tennis balls could the monster have thrown back out of the box?

Clearly the answer to this question depends on how the tennis balls are labeled. We will consider this situation and question for a few different labelings (positive integers, positive integers with repetition).

*This presentation is appropriate for undergraduates.

Special Session on the History of Mathematics I

1. *Raymond Pearl and the Logistic Curve*, Bob Fray, Furman University

In this talk I look at the history of using the logistic model to describe population growth. In particular, I discuss the crusade by Raymond Pearl to convince the world that the logistic model is a biological law (similar to Newton's Laws of Motion). He published many papers, several books, and gave many talks promoting his thesis. However, in the end he crashed. The logistic model can be understood by anyone who has had the first course in calculus; it is frequently used in texts as an example of an application.

2. *Queen Dido's Hide and the Minimal Arc-length problem in Calculus*, Wally Javier, Southern University-Baton Rouge

There are infinitely many functions $y = f(x)$ defined on the unit interval of the x- axis; non-negative there; $f(0) = f(1) = 0$; and encloses unit area. Among these functions is there one that has the least arc length? The answer becomes very accessible when the class of possible solutions is enlarged to the class of all plane curves joining the two endpoints of the unit interval---as a simple corollary of Queen Dido's Theorem, namely: Among all plane loops with same measure of arc-lenght L, the circle encloses the largest area. The solution is computed, and verification is done by comparison with other well known curves by TI-83- level arc-length computations.

3. *The Influence of Neighboring Scientists and Faculty on the Development of Mathematical Sciences at Clemson University*, T. Gil Proctor, Clemson University

The Department of Mathematics (later renamed Mathematical Sciences) began operation in 1892, began awarding bachelor Bachelor of Science concentration in mathematics after World War II, M. S. degrees in 1960 and Ph.D. degrees in 1965. This talk will be concerned with how faculty and scientists in other institutions aided this development. The intention is to talk about much more than the conferences surrounding the NSF grant "Alternatives in Mathematics in Higher Education". This grant, although enormously significant for the department's development, is already quite well-known.

4. *Understanding Mathematical Proof: The Four Color Problem and a Math Forum MidPoW*, Craig Bach, Drexel University

I regularly teach a course on the History and Philosophy of Mathematics for pre- and in-service teachers. The goals of the course are to deepen teachers' understanding of mathematical concepts and to provide them with a more powerful set of skills to address educable moments in their mathematics classrooms as well as enrich their teaching methods and lesson plans. One topic we regularly cover is the computer-assisted proof of the Four Color Problem, and just as regularly I find that the teachers have a difficult time understanding its significance (and relevance). However, everything turned around during the discussion of a simple middle grades problem of the week (MidPoW). In this presentation, I will discuss how the teachers' reactions to an easy "counting" problem mirrored the discussion surrounding the proof of the Four Color Problem, and in turn how that discussion deepened their understanding of the concept of a mathematical proof and improved their abilities to address a wider range of student responses to the MidPoW (i.e., helped improve their pedagogical skills).

Special Session on Integrating Applied Problems into the Undergraduate Curriculum I

1. *Using the Historical Development of Predator-Prey to Motivate Modeling*, Holly Hirst, Appalachian State University

After the Lotka-Volterra predator-Prey model was introduced in the 1920s, many scientists suggested improvements to incorporate more realism. The modifications proposed between the 1920s and the 1970s can be used very effectively to introduce students to the modeling process. This presentation will cover two strategies, one to use in a freshman liberal arts course and one to use with students who have had or are taking calculus.

2. *Mathematical Modeling of the Terror Bird*, William P. Fox, Francis Marion University

During an archaeology dig in Florida, a prehistoric femur bone was found. As the dig continued, many more bone fragments were found that allowed the paleontologists to recognize the dinosaur as a Titanus Walleri. The Titanus really lived about 2 million years ago on the oak and grass savannahs of what is now Florida. The scientists believe that the Titanus was a bizarre predatory bird, and it has been called the "Terror Bird". We present projects from college algebra and from higher mathematical modeling electives that can be used to advance the use of mathematical models and projects in mathematics courses.

3. *Applied Mathematics for Undergraduates at UT*, Suzanne Lenhart, University of Tennessee

At the University of Tennessee in Knoxville, applications of mathematics and student research projects are present in our coursework and in our Research Experiences for Undergraduates program. Two courses will be discussed, Mathematical Modeling and Industrial Mathematics. Some examples of applied research projects from the courses and from our REU program will be given. The structure of our REU program will be briefly discussed.

4. *National Computational Science Institute: Modeling in the Classroom*, Daniel Warner, Clemson University

The National Computational Science Institute has been funded in part by the NSF to provide workshops for undergraduate math and science faculty who are interested in exposing students to computation as a complement to the traditional research tools of theory and experimentation. This presentation will give an overview of the workshops, particularly emphasizing the role of mathematics and computer science in computational science.

Commercial Presentations

1. Houghton Mifflin MathSpace: Flexible, Integrated Electronic Learning Tools

Barbara Siry, Barbara_Siry@hmco.com, Kelly Huskey
In order to help faculty understand the best use for the technology supplements that accompany their HM textbooks, we would like to demonstrate the application of several of our core digital solutions. These include our free, online tutoring service entitled: Smarthinking; our online homework program called: eduspace; and our computerized testing program: HMTesting.
We also publish the Don LaTorre, John Kenelly and Iris Fetta text: Calculus Concepts 2e and Brief Calculus Concepts 2e and will be happy to discuss the websites and ancillaries that accompany these titles.

2. The Virtual Math Lab by Addison Wesley Publishing

Dinya Floyd, dinya.floyd@aw.com, Marty Mcdonald, marty.mcdonald@aw.com
Please join us for a demonstration of MyMathLab.com, an online teaching and learning tool designed to accompany all major Addison-Wesley mathematics textbooks. Enhanced by interactive Java applets, animations, and video and audio content, MyMathLab offers algorithmically generated tutorials and testing tracked in a nationally hosted course-management system.

3. Features of BCA, Brooks Cole Assessment

Trent Whatcott, trent.whatcott@thomsonlearning.com
We appreciate the opportunity to demonstrate Brooks Cole Assessment (BCA) at Clemson in March. BCA incorporates comprehensive diagnostic, testing, tutorial and course management and whole assessment methods. In our presentation, we will demonstrate to attendees:

- How to log on to BCA
- How to create new courses in an instructors own BCA page
- How to create gradebook categories such as:
    - Homework
    - Quizzes
    - Tests
    - Final Examinations
- Assigning percentage weights to each category
- Creating an algorithmically generated test with the integrated test wizard.

The features mentioned above will allow an instructor to provide their students with algorithmically generated quizzing, testing and homework, as well as offer students intuitive, feedback based tutorials that will shorten the line to their office door.

4. Texas Instruments

4:30-5:30 General Session II

Ronald Harshbarger
University of South Carolina Beaufort
RISKY BUSINESS: Investigating the Connection Between Mathematics and Business Concepts
Suppose a company is labor intensive, so that the monthly fixed cost for a product is $10,000 and the variable cost is $100 per unit, but installing modern equipment would reduce the variable cost to $50 per unit while increasing the fixed cost to $30,000. Installing the equipment increases its **operating leverage**, which means that once break-even is obtained for the product, the lower variable cost will result in more profit per unit produced and sold. But it will also increase the **business risk**, because many more units must be produced to obtain break-even. How many units must the company be confident of selling before they decide to take the business risk of installing new equipment? To investigate the relationship between operating leverage and business risk, students work collaboratively to investigate how different prices and different fixed costs affect break-even and the rate at which profit grows. To gain insight into the relationship between the operating leverage of choosing high fixed costs and the business risk of failing to sell enough units to reach break-even, students must model the total revenue for x units sold at price p, they must model the total cost for various fixed costs and variable costs, and they must determine how the break-even quantity is related to the price when fixed cost and variable cost is known.
This is one example of many projects used to link finite math and business calculus to business topics. Business and economics majors will likely have future careers that require reading for comprehension, problem solving skills, and the ability to analyze and interpret. In addition, they are likely to work in teams to solve problems in their future jobs. Thus these group projects require students to work collaboratively to solve problems that connect mathematics concepts and skills with business and economics applications. These projects are designed to show students an interesting, challenging and useful approach to mathematics that will help them in future courses and careers.

By giving Business and Economics Majors compelling applications, they will see that there is some reason for the mathematics to exist, they will be more interested, and they might even come to like mathematics in this context. The applications emphasize real problems rather than mathematical theory for its own sake. These applications involve critical thinking, complete solution, and interpretation, and require a written report. Data analysis, modeling, and technology are used in the projects so that the approach is refreshing and interesting to the students. Other examples of the projects in which the students investigate the connection between mathematics and the business concepts that they are studying include "A Graphical Approach to Optimization in Business," "Maximizing Apartment Rental Revenue," and "Loan Consolidation: Good News or Bad News?"

SATURDAY MARCH 22nd

9:00-10:00 General Session III,

John Baxley
Wake Forest University
The Adventure of Mathematical Ideas

I will discuss some results that have been obtained in the last six years in collaboration with students, both graduate and undergraduate, at Wake Forest. The existence and multiplicity of solutions of nonlinear boundary value problems for ordinary differential equations, the qualitative behavior of these solutions, and their numerical computation have all been objects of these efforts. I particularly want to illustrate the ongoing vitality of the ideas of elementary calculus in mathematical work.

10:20-12:20 Concurrent Sessions

Mathematics Teacher Development

1. *College Algebra Computer Lab - friend or Foe?* Cynthia Sikes and Deborah Evans, Georgia Southern University

Instructors' and students' reactions to a course-linked web-based lab for College Algebra students will be shared. Two groups of students will be compared - those who are repeating the course and are required to enroll in lab and those who volunteer to enroll in lab.

2. *Breaking the Cycle of Mediocrity: Developing a Profound Understanding of Fundamental Mathematics among Future Teachers*, Betsy Darken, University of Tennessee at Chattanooga

Mathematics education in this country is caught in a vicious cycle of mediocrity. Students who learn mathematics as a myriad of
mysterious rules and unconnected facts become teachers who pass on their lack of understanding to the next generation. The relative ignorance of American elementary and middle school teachers regarding the underlying principles of elementary mathematics has been repeatedly demonstrated in research studies. This problem has recently been addressed by the
Conference Board of the Mathematical Sciences in their publication, The Mathematical Education of Teachers. Teacher preparedness is the weakest link in achieving the ambitious goals of the National Council of Teachers of Mathematics to improve mathematics education in this country. The purpose of this talk is twofold: (1) to discuss how expectations of college mathematics departments regarding future K-8 teachers must be raised in order to help break the cycle of mediocrity, and (2) to present
results achieved in a course designed to raise future teachers' understanding of the principles of K-8 mathematics.

3. *An Open, Flexible, Collaborative Web Homework System*, Terry Walters and Stephen Kuhn, University of Tennessee at Chattanooga

We discuss the WHS (Web Homework System) a web-based instructional support system developed at the University of Kentucky. We present some applications we make of it: as a tool for providing auto-checked mathematics homework sets, as a tool for offering an on-line college "placement" test, and as an environment for a problem development course especially useful for future teachers. The system is designed to encourage sharing and collaboration and the source files for the homework problem sets can be written in Maple, Word, etc. The University of Kentucky makes WHS freely available for non-profit, educational use. (This work is supported by an ILI grant from NSF.)

4. *Successful and Unsuccessful Proposal Writing Efforts in the East Tennessee State University Mathematics Department*, Anant P. Godbole and Jeff Randall Knisley, East Tennessee State University

One of the key agendas for the East Tennessee State University Mathematics Department has, in recent years, been the aggressive procurement of external funding -- in an effort to launch major new initiatives; revitalize faculty members; and bring much needed indirect cost return dollars into the department. This talk will outline some of our successful

efforts since 2000, including
(i) NSF-REU/RET grants;
(ii) NSF-CCLI grants;
(iii) An NSF-CBMS grant;
(iv) A W.K. Kellogg Foundation grant to improve K-8 Mathematics teaching;
(v) An "Improving Teacher Quality" ("Eisenhower") grant; and
(vi) NSF-AASCU, NSF-CLT, and Exxon-Mobil grants for K-8 teacher preparation;
We will also describe major well-received (but unfunded) proposals including ones written for the NSF-GK12; NSF-STEP; NSF-MSP; and NSF-IPSE competitions. We have established collaborations with other universities and community colleges; seventeen school districts; and a children's science museum. Such collaborations will continue to play a key role in our future submissions.

5. *Using a Coteaching Module in a Mathematics Methods Class For Elementary Preservice Teachers: Reflections on Practice*, Lisa Carnell, High Point University

High Point University has begun using coteaching modules in its elementary education methods classes in the four content areas: mathematics, language arts/communication, social studies, and science. During the coteaching module, which lasts three weeks, a special education faculty member joins a general education faculty member in the classroom, and they model coteaching for the preservice teachers while at the same time providing instructional strategies for special learners. The integration of coteaching into the methods classes provides preservice teachers with a model of instructional delivery that they might be expected to use in their own classrooms in the future. In addition, since a special education faculty member rotates through each of the four methods classes for three week modules, the coteaching module provides the equivalent of twelve full weeks of instruction in teaching exceptional children in the content areas without adding hours to the major.

There are issues that need to be addressed when implementing coteaching as a method of instructional delivery. Expectations of students with respect to grading, finding planning time, deciding on which coteaching model to use, and dealing with personality differences all need to be discussed by the coteachers.

During my presentation I will discuss the models of coteaching, how coteaching has been integrated into my elementary mathematics methods class, and my reflections on the experience of coteaching with a special education faculty member.

6. *A Mixed Approach to Teaching Linear Algebra*, Skip Allis, Elon University

Our Linear Algebra mixes computational and theoretical topics to satisfy both mathematics majors and minors as well as physics and 3-2 engineering majors. This semester I am using a modified Moore style approach for the theoretical topics and a more traditional lecture style for the computational topics, running them is parallel on different days of the week. I will present student reactions and outcomes and discuss the results.

Differential Equations, Dynamical Systems & Numerical Methods

CONTINUOUS GAUSS-NEWTON-TYPE ALGORITHM FOR NONLINEAR ILL-POSED OPERATOR EQUATIONS WITH SIMULTANEOUS UPDATES OF THE REGULARIZED FRÉCHET DERIVATIVE

Alexandra Smirnova

Georgia State University

Let F be a nonlinear twice Fréchet differentiable map in a Hilbert space H and the equation

$$F(u) = 0, \quad F : H \to H, \tag{1}$$

be solvable (maybe nonuniquely). Assume that y is a solution to (1), and $F'(y)$ is not boundedly invertible. In this case solving (1) is an ill-posed nonlinear problem. A new approach to solving such a problem is presented. The approach is based on a construction of a dynamical system

$$\dot{u} = \phi(u,t), \quad u(0) = u_0, \quad \phi : H \times [0,+\infty) \to H. \tag{2}$$

with a unique global solution $u(t)$ for any $u_0 \in B(y,r)$ that tends to an element $u(\infty)$ (in the norm of H), and $F(u(\infty)) = 0$. If y is a unique solution to (1) in the ball $B(y,r)$, then $u(\infty) = y$. Otherwise $u(\infty)$ depends on the choice of u_0.

A novel continuously regularized Gauss-Newton-type method with simultaneous updates of the operator $[F'^*(u(t))F'(u(t)) + \varepsilon(t)I]^{-1}$ for solving equation (1) in a Hilbert space is investigated in the framework of the dynamical system approach. The method consists of solving the system of differential-operator equations:

$$\dot{u}(t) = -B(t)\big[F'^*(u(t))F(u(t)) + \varepsilon(t)(u(t) - u_0)\big], \tag{3}$$

$$\dot{B}(t) = -\big[(F'^*(u(t))F'(u(t)) + \varepsilon(t)I)B(t) - I\big], \tag{4}$$

$$u(0) = u_0 \in H, \quad B(0) \in L(H), \quad 0 < \varepsilon(t) \to 0 \quad \text{as} \quad t \to +\infty.$$

The convergence analysis is given. As a consequence of the convergence theorem the stability of process (3)-(4) towards noise in the operator F is proved, and the choice of an optimal regularization parameter (the stopping time), such that the method converges to a solution of (1) as the noise level tends to zero, is done.

An attractive feature of the proposed algorithm is the absence of the assumptions about the location of the spectrum of the linear operator $F'(y)$. The absence of such assumptions is made possible by a source-type condition.

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2. *Interactive Generation of Orbits in the Restricted Circular Planar Three-body Problem*, Jack R. Pace, Southern Polytechnic State University

The restricted circular planar three-body problem involves a system of two second-order differential equations in a rotating coordinate system. This system has no general (formula) solution, so numerical techniques are used. With the starting point A, ending point B, and time interval T between them specified, the differential equations can be solved numerically as a two- point boundary value problem by using the shooting method with adaptive stepsize control. The simple shooting method works well if A and B are fairly close together and T is small. Once the first trajectory is drawn, either endpoint, as well as the interval T, can be changed incrementally by the user. The trajectory is rapidly recomputed and redrawn, achieving a rubber-band-like effect. This allows a search for stable near-periodic orbits as well as orbits with other interesting properties. Once it is created, the solution trajectory from A to B can be allowed to progress forward through time to check for (numerical) near-periodicity and other properties of interest. The graphical display can also be switched between rotating and inertial coordinates, and the instantaneous Keplerian ellipse and orbital elements can be displayed graphically. After a brief introduction, a computer program will be presented which shows the above features, and a gallery of some interesting orbits will be shown.

3. *A Summary of Results Pertaining to Multicomponent, Viscoelastic Fluid Flow*, Will Miles, Clemson University

This work investigates the modeling of multicomponent, viscoelastic fluid flows. The governing equations are presented and the
existence of the solution to the approximating system is shown to exists. Also, an a priori error estimate between the true solution and the solution to the approximating system is given. The nonlinearities in the approximating system of equations are handled by lagging the velocity. The set of governing equations includes the imposition of a ``jump" condition which exists at a fluid-fluid interface. The interfacial tension forces which act along the interface are transformed into volumetric forces via a method similar to the *continuous surface force* model of Brackbill, et al. The numerical implementation uses a level set method to track the interface. The fluids involved are allowed to have both viscous and elastic characteristics. In the case of multicomponent fluid flows, the interfacial tension forces may play a significant role in morphological development. Higher values of the *coefficient of interfacial tension* (CIT) allow for less deformation of the minor phase while lower values of the CIT allow the minor phase to udergo
large deformation under appropriate mixing conditions. To date, most numerical implementations assume that the CIT, denoted by σ, is constant. In the cases in which σ is allowed to vary, the surface gradient of σ, $\nabla_s\sigma$, which is present in the jump condition at the interface, is neglected. The model implemented in this work makes no such assumption. The CIT is allowed to depend on temperature. Thus, σ varies spatially and the surface gradient $\nabla_s\sigma$ is incorporated. These investigations are motivated by chaotic mixing processes implemented by scientists in the *Center for Advanced Engineering Fibers and Films* at Clemson University.

4. *Regularisation and control of self-focusing in the 2D cubic Schrödinger equation by attractive potentials*, Brenton leMesurier, College of Charleston, Peter Christiansen, Technical University of Denmark, Yuri Gaididei, Bogolyubov Institute for Theoretical Physics, Ukraine, Jens Juul Rasmussen, Risø National Laboratory, Denmark

The effect of attractive linear potentials on self-focusing in waves modelled by the Nonlinear Schrö dinger equation is considered, such as arise in models of optical fibres with narrow cores of different refractive index or wave-guides induced in a nonlinear medium by another beam, in models of behaviour near impurity in two-dimensional molecular structures and in the Gross-Pitaevskii model of Bose-Einstein condensates.

It is shown that adding an attractive potential to the cubic {\schr} in critical dimension one can prevent both singular collapse and dispersion that are generic in the CSE, and can lead to a stable oscillating beam.

This is seen to involve a splitting of the beam into an inner part that is oscillatory and of sub-critical power, and an outer dispersing part.

Analysis is given in terms of rate competition between the linear and nonlinear focusing effects, radiation losses and known stable periodic behaviour of certain solutions in the presence of attractive potentials.

5. *Optimal Harvesting in an Integro-difference Population Model*, Hem Raj Joshi, Suzanne Lenhart and Holly Gaff , University of Tennessee

We consider the harvest of a certain proportion of a population that is modeled by an integro-difference equation. This model is discrete in time and continuous in the space variable. The dispersal of the population is modeled by a integral of the population density against a kernel function. The control is the harvest, and the goal is to maximize the profit. The optimal control is characterized by introducing an adjoint function. Numerical results and interpretations are given for four different kernels.

6. *Summing Formal Power Series Solutions to Advanced and Delayed Differential Equations*, David W. Pravica and Michael J. Spurr, East Carolina University

Analytic advanced and delayed differential equations, which are defined in a neighborhood of a point, generate formal solutions in terms of power series. These formal solutions can be extended to solutions which are analytic on a sector domain with vertex at the initial point. New Laplace-Borel kernels are introduced and extended, and their decay rates are estimated for a class of Poincar\'e ranks. These are applied to generalized Gevrey series. Conditions on formal series solutions to advanced and delayed differential equations with real valued coefficients are given that provide for analytic solutions convergent on a sector. Examples are discussed, with applications given to advanced differential equations and to the growth rate of the Borel Transform of the formal solution. This talk will be accessible to students who may have seen a Laplace transform and have a knowledge of some complex analysis.

Statistics & Probability

1. *Needed: A Standard Measure for Comparing Distributions*, James Kropa, Southern Polytechnic State University

How well does distribution {a} compare with distribution {p}, where {a} represents the actual performance of something planned, {p} ? Using known concepts, a score will be presented which is a natural extension of percentages. This score grades how well the two distributions compare. The score can screen the performance of hundreds of business locations so the worst performing can be investigated.

2. *The Multivariate Local Time Intensities of Regenerative Sets*, Hussain Elalaoui-Talibi, Tuskegee University.

Let M be a regenerative set on the nonnegative real line with local time T. We will show that under some mild regularity condition, the n-dimensional multivarite intensities E(TXTX...T) are absolutely continuous with respect to the n-dimensional Lebesgue measure, and we will give a precise description of the continuity set of the associated densities.

3. *Half Way Through* e^x, Donald Francis Young, Southern Polytechnic State University

Several questions in probability and statistics lead naturally to asking the following: For any positive integer n, at what point in the Maclaurin series for e^n does the partial sum first exceed $(1/2)e^n$? One way of resolving this makes use of a series of problems in Polya and Szego's *Problems and Theorems in Analysis*.

4. *Inequalities for Renewal-Type Integrals with Applications* , Broderick O. Oluyede, Georgia Southern University

Renewal-type integral equations are useful in many contexts in applied stochastic processes, especially in the study of replacement problems in reliability and survival analysis. In this note, inequalities and bounds for renewal-type integrals are presented. Relations for renewal-type integrals of the ruin probablilty are also established and presented. Some inequalities, bounds, and comparisons for weighted models are established.

5. *The Singled Out Game*, Kennan Shelton, Rhodes College

4. All of the Maple procedures used are sufficiently elementary that students can learn and use them with only a minimum of instruction in the use of Maple

2. *Visualization of an affine transformation*, Subhash Saxena, Coastal Carolina University

This presentation will deal with the latest technology, which enables us to visualize and teach "affine transformations" in a manner that could not be done earlier. With the help of this technology, many topics can be taught effectively in Geometry classrooms. It enables us to dissect affine transformations, which will make sense to an "average" student.

3. *Introductory Analysis: Synthesizing R, R^n, Metric Spaces and Topological Spaces*, Robert Gardner, East Tennessee State University

All to often, undergraduate math majors get a thorough exposure to the structure of R (or maybe R^n), but are left entirely unexposed to the same concepts (eg. "open," "closed," "compact," "connected," "limit," and "continuity") in more general settings, such as metric spaces and topological spaces. This is unfortunate, since in most instances, it is possible to extend many of the results from R to R^n, metric spaces and topological spaces, with little to no revision (provided the relevant definitions are introduced). The purpose of this presentation is to show how a standard introductory real analysis class can be used, with minimal interruption, to extend many results from R to these more general settings.

4. *An Online Multivariable Calculus Course*, Jeff Knisley, East Tennessee State University

For the past 3 years, we have been teaching third semester Calculus at East Tennessee State University based on an online textbook. In this talk, we discuss the impact of this course on the students, the resulting transformation in the curriculum, and the possibilities for this type of instruction in other courses.

5. *Title: Summing k-th powers of consecutive positive integers: an elementary and generalizable approach for the Calculus I classroom*, Gregory M. Boudreaux, University of North Carolina at Asheville

After using the definition to evaluate the definite integral of a polynomial function, the calculus student fully appreciates the power
of the Fundamental Theorem of Calculus. However, this algebraically demanding evaluation depends on knowing the closed form of the sum of the k-th powers of the first n positive integers. Existing methods of calculating these closed forms are either too complicated to be introduced in a typical Calculus I classroom or do not generalize to all values of k. Here, a method is presented that overcomes these two difficulties.

6. *Addressing the Issue of Retention of Mathematics Majors: Seminar for Freshmen and New Mathematics Majors. Preliminary Report*, Patricia Shelton, Janis Oldham, North Carolina A&T State University

The course guides and encourages proper mathematics study habits, and develops an
informed mathematics major who will be prepared to move through his or her curriculum. Seminar topics include: how to study mathematics, ethics/academic honesty, introduction to higher reasoning, technology instruction, mathematics careers and their relation to program selection, undergraduate research, pre-professional information, and more. The course is separate from the university's freshman orientation course, Freshman Studies 101. The course has been run twice with class sizes 14 and 31. The first subsequent sophomore Mathematical Logic and Proof Techniques courses had class size of 27.

Undergraduate Student Papers III

1. *Homothetic Triangles with Coincident Euler and Nagel Lines*, Robert Davis, Elon University

The Euler and Nagel lines of a triangle are well known in triangle geometry. This talk discusses similar triangles with parallel sides created by trisecting the sides of a triangle. Using known triangle centers, including the median point, the orthocenter, the circumcenter, and the incenter, it will be shown that these triangles have coincident Euler and Nagel lines. Using these homothetic triangles, we can create an infinite number of points on the Euler and Nagel lines that converge to the median point of a triangle in specific ratios.

2. *Fibonacci Vectors*, Ena Salter, Georgia Southern University

We consider vectors in two and three dimensions whose entries are Fibonacci numbers and find many interesting properties.

3. *Colors, Clusters and Approximating the SVD*, Nick Orlowski, NCSU

Clustering and SVD approximating are methods that can be used to quickly analyze a given data matrix. These matrices are usually statistical in nature, be they a collection of documents or a collection of student scores. The above methods can be used to extract main ideas from the data in order to simplify them and can also be used to _lter noise. It is sometimes di_cult to compare clustering and SVD approximations. Their methods are related, but as the number of dimensions increases, the numerical results can become harder to interpret. In order to `visualize' the data we can map each vector to a color, a linear combination of the chosen basis colors. For example, a matrix whose columns are 3-dimensional data vectors could be mapped to the RGB (red, green, blue) space. This idea can be extended to higher-dimensional matrices, where you would add a color for each dimension you wished to represent. For example for 6-

dimensional data vectors we could choose Red, Yellow, Magenta, Cyan, Blue, Green as the basis.

4. *Normalized Circular Bernstein-Bezier Curves*, Mary Beth Cole, Samford University

Bernstein-Bezier polynomials are established tools for constructing splines and play an important role in fitting functions to data and in approximating complicated functions. In a recent paper, Alfeld, Neamtu, and Schumaker introduced Circular Bernstein-Bezier (CBB-) Polynomials, analogs of the well-known Bernstein-Bezier polynomials.

We explore an alternative trigonometric analog of the familiar Bernstein basis polynomials, defined in the following way: Let $d > 0$, and let θ_1 and θ_2 be such that $0 < \theta_2 - \theta_1 < \pi$, and let $\theta_1 \le \theta \le \theta_2$. Then the normalized circular Bernstein basis polynomials of degree d on $[\theta_1, \theta_2]$ are

$$T_i^d(\theta) := \binom{d}{i} \frac{\sin^{d-i}\left(\frac{\theta_2-\theta}{2}\right)\cos^{d-i}\left(\frac{\theta-\theta_1}{2}\right)\sin^{i}\left(\frac{\theta-\theta_1}{2}\right)\cos^{i}\left(\frac{\theta_2-\theta}{2}\right)}{\sin^{d}\left(\frac{\theta_2-\theta_1}{2}\right)}, \quad i = 0,\ldots,d.$$

These polynomials are designated "normalized" since, unlike the CBB-basis functions,

$$\sum_{i=0}^{d} T_i^d(\theta) = 1, \quad \theta \in [\theta_1, \theta_2].$$

Curves of the form

$$\sum_{i=0}^{d} C_i T_i^d(\theta), \quad C_i \in \mathrm{R}^2, \quad \theta \in [\theta_1, \theta_2].$$

are called Normalized Circular Bernstein-Bezier (NCBB-) curves of degree d. We will present a general theory of the NCBB-curves paralleling the familiar results in the polynomial Bernstein-Bezier case, including derivative formulae, a deCastlejau algorithm, a degree-raising formula, subdivision, and blossoming.

Undergraduate Student Papers IV

1. *Random Growth Of Caterpillar Graphs*, Gabriel Zimmer, East Tennessee State University

Consider a Caterpillar graph grown in the following manner: start with a single vertex, then let a vertex be added to the graph at each time interval along with a corresponding "segment". We will define the probability distribution at each time interval so that further segments are placed so that either a segment (i) is a leg of an existing spinal vertex, or (ii) is a new spinal vertex, with equal probability. For example, if such a caterpillar has spinal vertices {A,B} at time t then the segment "grown" at time t+1 is either a "leg" of A, a leg of B, or a new spinal vertex C with equal probability 1/3. Let the (random) length of the caterpillar on n vertices be denoted L_n. We prove that $P(L_n = k) = S(n,k)/B_n$, where $S(n,k)$ is the k^{th} Stirling number of the second kind, and B_n is the n^{th} Bell number. We provide exact formulae for $E(L_n)$ and $Var(L_n)$, study the asymptotic values of these quantities and explore the limiting distribution of the caterpillar's length.

2. *Flipping Geometry*, Shaun Lynott, Elon University

Through the study of geometry and complex numbers, there are many things to discover about polygons and the planar positions of their centers. Much of what will be discussed here revolves around the operation of flipping polygons, which is a reflection of a polygon and its center about one of its sides. The density of flipping centers in the complex plane will also be discussed.

3. *Upside-Down Numbers…Upside-Down*, Chaska Mendoza, Elon University

Throughout the music world, there are pieces of music that sound the same whether played by rotating the paper 180° and playing the music right-side up or upside-down. In the article, *Bach, 5465, and Upside-Down Numbers*, the authors examine upside-down numbers. In this presentation, we will define what it means for a number to be upside-down and we will determine how many upside-down numbers of length *d* exist. To further the discussion, we will also investigate upside-down numbers whose complements sum to *n*, where $2 \le n \le 18$.

4. *A Rate Dependent Preisach Operator for Modeling A Piezoelectric Stack Actuator*, Jeremy Poling, Ferrum College

A premise of the Preisach operator is that it is independent of the input rate. In modeling physical systems it therefore ignores the true fact that the response of the system is typically dependent on the input rate. The focus of this paper will be to develop a rate dependent hysteresis model for a piezoelectric stack actuator based on a Preisach type operator. Like other Preisach type operators, this one will model the hysteresis behavior as a cumulative sum of elementary operators. However in this case, the elementary operators are modeled as elementary resistor-capacitor circuits and therefore will exhibit a rate dependent response. The cumulative dynamics of these elementary operators allows us to develop a rate dependent hysteresis operator. For application purposes, an approximation is created and used to develop a simulation of an actuator. We demonstrate the feasibility of this approach by comparing the simulation output to experimental data collected at different rates on an unloaded piezoelectric stack actuator.

Graduate Student Contributed Papers II

1. *Orthogonal quadruple systems and 3-frames*, Brian Muse, Auburn University

Orthogonal quadruple systems are defined and investigated. Orthogonal quadruple systems with an additional nesting property are shown to provide a new construction of frames F(3,4,n{2\). Constructions for these systems are provided.

2. *Maximal Sets of Hamilton Cycles*, Sasha Logan, Auburn University

A set S of edge-disjoint hamilton cycles in a graph G is said to be maximal if the hamilton cycles in S form a subgraph H of G such that G-E(H) has no hamilton cycle. The spectrum of a graph G is the set of integers m such that G contains a maximal set of m edge-disjoint hamilton cycles. This spectrum has previously been determined for all complete graphs, all complete bipartite
graphs, and most complete multipartite graphs. In this paper we look at one of the two unsolved cases for complete multipartite graphs.

3. *Periodic Solutions in an Elastoplastic Model for Granular Flow*, Bob Wieman, NC State University

The flow of granular materials is physically important in many industrial and agricultural applications, but our understanding of the models describing granular flow is still limited. A one-dimensional continuum model has been developed by Schaeffer, Shearer, and Witelski which retains interesting features of granular flow, such as shear band solutions. The elastoplastic form of this model generates a periodic solution, which consists of elastic and plastic waves, as well as transitions between the elastic and plastic states. We exhibit this periodic solution and describe its components. Most continuum models of granular flow assume plastic behavior; the dramatic change in behavior when elasticity is introduced suggests that the elastoplastic nature of granular materials may have a more significant influence on their behavior than previously assumed.

4. *Performance based decisions under uncertainty for complex systems*, S. Samson, Clemson University

A decision-making methodology for complex systems performing under uncertainty is applied to the problem of choosing a best candidate for a position at a university. Performance of a candidate is viewed as a two-level complex system whose components interact and decisions on the components are made under uncertainties. The methodology uses stochastic analysis and multicriteria analysis, and is composed of two stages, top-down modeling and bottom-up decision making. In the modeling stage, the relationships between components are developed and computational models are selected based on decision maker's risk assessment. Stochastic linearization representations are used to model risk associated with the candidates. In the decision stage, efficient candidates are first identified among all the candidates applying for the position. The candidates are compared based on their performance represented by response surfaces or functions with respect to independent variables defining uncertainties. A preferred candidate is selected from among the efficient candidates based on decision-maker's preferences.

5. *The Ship Captain's Problem*, Sarah Holliday, Auburn University

We shall resolvably decompose the complete multipartite graph with a one factor removed, with r parts each of size a into uniform cycles of length k. We shall give necessary conditions and some constructions.

6. *Green's Function for an Equivalent Cable Model*, Scott La Voie, East Tennessee State University

In this talk we develop the Green's Function for a tapered equivalent cylinder model of dendritic electrical propagation. We
then use the Green's Function to develop a Carleman linear embedding scheme which is used to estimate the effects of a nonlinear ion channel hot-spot on the tapered cylinder solution.

Special Session on Discrete Mathematics II

SATURDAY, March 22, 10:20 - 10:45

Domination in Triangulated Chessboard Graphs

Charles Wallis, cwallis@wpoff.wcu.edu, Western Carolina University

The rook's graph, in which the vertices consist of the n2 positions of an n x n chessboard and adjacencies represent the possible moves of a rook, corresponds to the graph of the L2 association scheme. In a similar way, chessboard-like graphs can be defined based upon the relation known as the triangular association scheme; we designate these graphs triangulated chessboard graphs. In this talk, the values of several domination parameters of triangulated chessboard graphs are determined and compared to the known domination parameters of ordinary chessboard graphs. This is joint work with Renu Laskar at Clemson University and Alice McRae at Appalachian State University.

SATURDAY, March 22, 10:50-11:15

Total k-Subdominating Functions on Graphs

Laura Harris, University of Natal, Johannes H. Hattingh*, Georgia State University, and Michael A. Henning, University of Natal

A two-valued function f defined on the vertices of a graph $G = (V, E)$, $f: V \to \{-1, 1\}$, is an opinion function. For each vertex v of G, the vote of v is the sum of the function values of f over the open neighborhood of v. A total k-subdominating function (*TkSF*) of a graph G is an opinion function for which at least k of the vertices have a vote value of at least one. The total k-subdomination number, $\gamma_{ks}^t(G)$, of G is the minimum value of $f(V)$ over all TkSF's of G where $f(V)$ denotes the sum of the function values assigned to the vertices under f. We give a lower bound on $\gamma_{ks}^t(G)$ in terms of the minimum degree, maximum degree and the order of G. A lower bound on $\gamma_{ks}^t(G)$ in terms of the degree sequence of G is given. Lower and upper bounds on $\gamma_{ks}^t(G)$ for a tree G are presented.

SATURDAY, MARCH 22 11:20 - 11:45

Locally Restricted Compositions

Rodney Canfield, erc@cs.uga.edu, UGa (joint work with Ed Bender)

Compositions $n = a_1 + a_2 + \cdots$, $a_k > 0$, have been studied classically. More recently, compositions with the local restriction $a_k \neq a_{k+1}$ (Carlitz compositions) have been studied by various authors. We consider the compositions with more general local-nonequality restrictions, including multiline compositions. We obtain recursions, bounds on growth rate, and other properties of a randomly selected restricted composition.

SATURDAY, March 22, 11:50 - 12:15

Long Cycles in 3-connected Graphs

Guantao Chen, gchen@cs.gsu.edu, Georgia State University

In 1931, Whitney proved that every 4-connected planar triangulation contains a hamiltonian cycle. In 1956, Tuttle generalized to all 4-connected planar graphs. In 1963, Moon and Moser conjectured that every 3-connected planar graph on n vertices has cycle of length

at least $cn^{\log_3 2}$, where c is a universal constant. In 1973, Grunbaum and Walther made the same conjecture for 3-connected cubic planar graphs. Chen and Yu recently established the Moon-Moser conjecture. A 3-connected graph with at least 6 vertices is planar if, and only if, it does not contain $K_{3,3}$ as a minor. Thomas conjectured that for any positive integer t there exist two positive real numbers a and b such that every 3-connected graph on n vertices without a $K_{3,t}$-minor contains a cycle of length at least an^b. In a joint work with Sheppardson, Yu, and Zang, we established the conjecture. We will outline the proof and discuss related problems during this talk.

Special Session on the History of Mathematics II

1. *Euclid's Elements, How Should We Approach the Text*, John Poole, Furman University

Some have described The Elements as monotonous and inhuman, while others have called it a splendid axiomatic development of mathematical ideas. I will argue that within the pages of this 2300 year-old text we find accessible and beautiful drama that, properly presented, can excite and please audiences even today. It is an educator's gold mine of incredible richness which can be used at both the high school and college
levels. I have prepared an interactive presentation of Books I - IV of The Elements which you can find at http://math.furman.edu/~jpoole/euclidselements/euclid.htm.

2. *Transformational Geometry in Art and Architecture of Pre-Columbian Latin America*, Elizabeth C. Rogers, Piedmont College

Beginning with a brief illustrated discussion of symmetries of today's transformational geometry used in the Pre-Columbian period, we explore the arts and architecture of ancient Latin American cultures.

From the ancient civilizations of Peru to the Aztecs of Mexico, use of transformational geometry is evident in the design and decoration of vessels, structures, and textiles. The discussion of each civilization's art and architecture is accompanied by representative photographs.

Beginning in the northwestern portion of South America with the Chavin, we follow the development of successive civilizations of the Andean area. The Moche and Nazca were masters at pottery and contributed to the developing tradition of Andean textiles. The people of Huari are remembered for both their complex system of subterranean tunnels and their artwork-particularly textiles.

One of the largest "city-state" type settlements to exist in the Andes region was that of the Chimu. Examples of Chimu design exist today in the form of pottery and architecture. Prior to the arrival of the conquistadors, Inca warriors conquered other groups and absorbed their cultures including pottery, metallurgy, and construction.

Looking at civilizations further north, the Zapotec and Toltec in Mexico show extensive use of transformational geometry in architecture as evidenced at Mitla and Tula respectively. We conclude with the Mayans who had well-developed art forms, including textile weaving and pottery plus detailed methods of carving and creating raised decorations using stucco.

3. *H.S.M. Coxeter: His Life and his Romance with Symmetry*, F. Arthur Sherk, University of Toronto and Clemson University.

H.S.M. Coxeter has been producing, writing and teaching Geometry for 80 years. His love of symmetry has led him many times to study regular polytopes and generalizations thereof. Results of this study are his many contributions to the theory of Discrete Groups. In this talk, we sketch out a biography of Coxeter's early years, combined with an introduction to his work on symmetry.

4. *History of Topology*, Artur Gorka, Clemson University

Topology is one of the most important developments in modern mathematics. It is concerned with those properties of a space (such as the number of "pieces" or the no. of "holes", etc) that remain invariant under continuous transformations. Its formal development started only at the end of the XIX century but some ideas where already perceived by Euler. Today topology lends its ideas and tools to both analysis and algebra. In this talk we'll present the historical development of the topology and some of its famous theorems.

Special Session on Integrating Applied Problems into the Undergraduate Curriculum II

1. *Internships for Undergraduates: Opportunities and Resources*, Angela B. Shiftlet, Wofford College

Internships can expose students to many new ideas, techniques, and applications that can greatly enhance their knowledge of computational science and make their classroom education more meaningful. Realizing the importance of such experiences, Wofford College's undergraduate Emphasis in Computational Science (http://www.wofford.edu/ecs) requires a summer internship. However, obtaining meaningful internships can be challenging. Committed to computational science education, the Krell Institute offers a website on internships (http://www.krellinst.org/ucse/internships.html) that provides valuable advice and links to other resources.

2. *Environmental Mathematics*, Bernard A. Fusaro, Florida State University

This is a Calculus-free approach to modeling natural systems. The target system is first represented by an energy diagram. This diagram is the basis for a qualitative Energy vs Time solution graph & for the flow equation (a DE in disguise). The flow equation is solved with a calculator or spread sheet, and the numerical output is used to plot a conventional Energy vs Time graph.

3. *A Second Year Course on an Introduction to Applied Mathematics*, R. E. White, North Carolina State University

This three-credit course is survey of applications of mathematics, which is suitable for students who have taken multivariable calculus. It enables the student to formulate a cohesive plan of study for the third and fourth years. Mathematics education majors find the variety of applications and a sampling of teaching styles to be very interesting. Also, perspective majors in pure or applied mathematics find this to be a good survey of applied mathematics beyond calculus. In the spring of 2002 there were five three-week modules on: heat and pollutant transfer (R. E. White), acoustic waves and boundary conditions (H. T. Tran), cryptographic schemes (E. Stitzinger), biological applications (S.

Lubkin) and modeling of random phenomena (J-P. Fouque). Each module serves as motivation for future course work and related academic activities.

4. *Solving a Social Problem with the Transportation Algorithm*, Laurie Heyer, Davidson College

The transportation problem is a classical operations research problem of shipping goods from factories to warehouses at minimum cost. As a special case of linear programming, the transportation algorithm is an appropriate topic in several courses, including mathematical modeling. I will describe an application in which students are assigned to social groups on campus at minimum overall dissatisfaction.

Undergraduate Student Posters

1. *SpaceShips: A look at video games and student motivation*, Susan Edwards, Meredith College

This project explores the effect that programming video games has on improving student motivation in an attempt to study the usefulness of game programming as an educational tool. In the past, students exit introductory programming courses feeling overwhelmed or unchallenged. By first introducing programming logic with graphical representations of source code, SpaceShips intends to provide a gentle transition into the Java programming language. SpaceShips simulates a spaceship battle by allowing students to pre-program a spaceship with some simple methods: shoot, move, and turn. From interactions with this program, students will gain an understanding of how to reason logically through programs, read and understand methods, and formulate their own methods. Benefits of the program include exposure to classes and class structure, ability to think about outside factors in relation to their programs, and ability to work in teams.

2. *Checking for Substructures in Graphs of Fixed Pathwidth*, Jarrett Walsh, Armstrong Atlantic State University

Using a previously developed algebraic representation system for graphs of bounded pathwidth, efficient membership algorithms are built for subgraph, minor and topological-minor partial order operations. Semi-automatic procedures for these problems are presented through numerous explanatory figures and source code. An enumeration scheme for building the membership automata for fixed bounded graph substructures will also be introduced.

3. *A small cover for convex unit arcs*, Joe Johnson, East Tennessee State University

We describe a compact convex plane set of area about 0.2466 that contains a congruent copy of each convex plane arc of unit length, a reduction of about 1.1 percent from the smallest such set previously known.

4. *An Examination of a Queuing Model*, Evelyn Thomas, Spelman College

In this study, I examined the waiting-line process, as well as its practical application in the Birth-Death Process. I developed population-dependent formulas that give the probability that a population is of size k at a particular time t, for when k*1 and k=0. The waiting-line procedure accounts for the probability of a birth or death in a given time interval. In addition, the waiting-line process mirrors the Poisson distribution, under the assumption that births and deaths are independent events, and in a given interval, at most one death and one birth may occur.

5. *A Comparison of Centrality Estimators*, Jamie McCreary, Tennessee Tech

The measure of central tendency is the most commonly used tool in statistical data analysis. The ability to determine an ``average" provides a way to locate data centrality. Central tendency is usually determined by one of three methods. One can calculate the mean, median or midrange of a sample set. However, does the best method to determine the central point of a distribution vary with the types of distributions involved? In this paper we attempt to determine which methods are best used for several different distributions. Specifically we will examine the Normal, Uniform, and Cauchy distributions and discuss why our results were not what we expected.

6. *The Parameter Space for the Iteration of Cubic Polynomials*, Jack Senechal, University of North Carolina at Asheville

The cubic connected locus is a four dimensional analog of the Mandelbrot set for the iteration of cubic polynomials. This poster presents some images of the cubic connected locus generated by a java applet written by the presenter.

7. *The Dynamics of $F_c(x) = cx(1-x)$*, Tammeca Rochester, Spelman College

We study the periodic point structure of the logistic family of functions $F_c(x)=cx(1-x)$, where c is a positive constant. These functions arise in the study of population growth.

8. *The Relationship Between Primes and Perfect Squares*, Charles N Glover, Morehouse College

Prime numbers have fascinated mathematicians for centuries because these numbers are an important part of mathematics. The fundamental theorem of arithmetic states that every natural number is either prime or can be expressed as the product of primes. This led mathematicians to search for methods to express this prime factorization of numbers. One of the oldest and most used methods is the division algorithm. Factors of a number are found by performing divisions using a sequence of increasing numbers. A second means of factoring an odd number, n, is through difference of two perfect squares, that is n = a2 - b2 = (a + b)(a - b), where a + b and a –b are two of the factors of n. If a + b = n and a - b = 1 are the only a, b such that the equation n = a2 - b2 is true, then n is prime. This method of factoring established a relationship between prime numbers and perfect squares. The goal of my research is to gain

knowledge on prime numbers by exploring their relationship with perfect squares.

9. *The Hamming (7,4) Code*, Aminah Perkins, and Andrea Warren, Spelman College

Errors are likely to occur when transmitting data across any channel. R.W. Hamming of Bell Telephone Laboratories developed an error correcting code, which we now call the Hamming (7,4) code, in the late 1940s. This code can be viewed as an application of Z_2. It allows messages to be corrected if an error exists. The Hamming (7,4) code is a linear code of 7 characters, which consists of 4 information characters with 3-error correcting parity checks attached to the end. In this presentation, we will explain the mathematics of this code and perform a demonstration of how it is used to encode and decode messages. In order to demonstrate the use of the (7,4) Hamming Code, we will demonstrate a card trick based on this code,

10. *Error-Correcting Codes*, Hatshepsitu Tull and Kamilah Mooney, Spelman College

When messages are transmitted, errors may occur in the messages, thus giving incorrect information when it is received. Error-correcting codes reduce the amount of incorrect information received. Since the development of modern electronic computing, these error-correcting and error detecting codes have become essential for the success of numerous businesses and their applications are endless. For example, the ISBN number on books and the Universal Product Code (UPC) on packaged goods use methods of error detecting to ensure that the least amount of errors occur when transactions are processed. Another application involves satellite transmissions. Bose, Chaudhuri, and Hoquengham codes (BCH codes) are multiple error-correcting codes that use finite fields. We will demonstrate how to construct finite fields, encode messages to be transmitted, and decode the received messages.

11. *Missile Launching: A Simplified Statistical Model*, Jamie Chatman, Spelman College

This study uses information from the undergraduate Mathematical Statistics course to obtain a statistical model for missile launching. The model uses basic probability theory and independent repeated Bernoulli trials. We estimate the number of days it will take to launch three missiles from two launching pads. Possible delays that may occur during countdown are taken into account.

12. *An Algorithm for Counting Finite Topologies*, Sean Rae, Winthrop University

There is currently no known formula for calculating the number of possible topologies on a given set. The binary sequence for the order of a set and the number of topologies that can be constructed from that set is known up to order 16 and no solution other than brute force calculation is known to extend to 17 and beyond. I propose a generalized algorithm for computing this sequence, given a user-inputted set order.

13. *A Relationship Between General and Second Order Linear Recurrences*, Daniel C. Morton, Wake Forest University

We determine that the bounds $\{B_n\}$ of a general linear recurrence with coefficients restricted to the range [-A, 0] (where A>0) follow a second order linear recurrence beyond a specific N. Said value of N is determined by where in the interval $[0, \infty)$ A is chosen from. Specifically, we have one value for N, when A is greater than 1, and multiple values for A less than 1. We will also discuss values of B_n when $n<N$. The results lead to interesting implications regarding coefficients of reciprocals of formal power series and solutions to triangular matrix equations.

14. *The Effect of Academic Achievement on Self-Esteem of an Early Adolescent*, Christy DeWees, Meredith College

The middle school years are a very fragile time in a person's life; childhood is being left behind while adulthood is prematurely emerging. One important aspect of early adolescence is self-esteem. Previous studies have indicated many influences on self-esteem. However, information about the effect of academic achievement on self-esteem is limited. In this study, self-esteem and academic achievement in mathematics and language arts were monitored in an experimental group with twelve subjects and a non-equivalent control group with fifteen subjects. The experimental group attended a weekly tutorial session to aid academic achievement. The data indicated that early adolescents' self-esteem was not significantly affected by variances in academic achievement. The only subgroup with a slightly significant correlation was the school self-esteem subgroup that increased in mathematics academic achievement.

15. *Mathematical Models of HIV Disease Pathogenesis*, Karen Herman, NC A&T State University

My presentation will be covering an ongoing research project involving mathematical and biological concepts. The research entails using previously discovered differential equations to develop a mathematical model to represent the HIV-1 virus affects on the human immune system. The research will mathematically conclude the HIV-1 virus affects without medical treatment and with medical treatment on the rate of production of CD4 positive T lymphocytes cells, the rate at which the CD4 positive T lymphocytes cells become infected, and the rate at which the HIV-1 virus reproduces. Only medical treatments involving protease inhibitors and reverse transcriptase inhibitors will be covered in this presentation. The conclusions involving no medical treatment will be graphically represented using *Maple*. One goal for this research project is to construct an interactive program capable of graphically reproducing results dependent on the user's specification of parameters. Another goal for this research project is to compare the genetic mutation rates of the HIV-1 virus due to medical treatments to the genetic mutation rates of the HIV-1 virus without medical treatment.

The final goal of this research project to extend the medical treatments to include integrase inhibitors.

16. *On the Difference Equation:* $X_{n+1} = p + X_{n-1}/X_n$, Allison Carter, Coastal Carolina University

The equation that we are examining is $X_{n+1} = p + X_{n-1}/X_n$, n = 0, 1, 2, . . .with initial conditions $X_{-1}, X_0 > 0$, $p \geq 0$. We are interested in the long-term behavior of solutions of this equation. In particular, we are interested in proving the following conjecture: "There exists a solution that lies above the equilibrium forever." We will present some results related to proving this conjecture and convince the audience that the conjecture is true.

17. *Numerical Solutions to the 1-D Schrodinger Equation,* Shaun Wood, College of Charleston

Because of the difficulty solving the Schrodinger equation analytically, I have written a program that uses linear algebra algorithms to generate numerical solutions to the 1-D Schrodinger equation. The accuracy of these solutions has been checked by comparing them to analytic solutions when they exist.

18. *KdV 2-Solitons,* Kevin Young, College of Charleston

The discovery of solitons -- non-linear waves with striking particle-like behavior -- in the late 1960's was just the beginning of an area of research that remains active today. This talk will reconsider one of the earliest significant results: the behavior of a "two-humped" initial condition which can be interpreted as the "collision" of two solitons. In particular, we will determine the four lines which are asymptotic to the motion of the peaks. One interesting feature of this talk is that we will be able to derive these asymptotes using the formalism of tau-functions, which was unknown when this result was first determined by other researchers over 30 years ago and greatly simplifies the computations. Moreover, we will use the results to answer the following question: "Can one distinguish a collision of solitons from a collision of classical billiard balls using only these asymptotes?"

19. *Applications of Algebra to Knot Theory*, Nancy Lin, University of Tennessee REU

We investigate the Alexander Polynomial of a two-bridge link, and obtained a new geometric proof of the previously known result that the Alexander polynomial of a two-bridged link is alternating. The proof uses a lemma stating that the group of a link admits a representation of a certain special form. Two different proofs of the lemma are given -- one using a plat representation and the other, Schubert's two-bridged "maze" representation.

20. *A Predator-Prey Model with Disease Dynamics*, Chris Flake, North Carolina State University and University of Nebraska-Lincoln REU

We propose a model to describe the interaction between a diseased fish population and their predators. Analysis of the system is performed to determine the stability of equilibrium points for a large range of parameter values. The existence and uniqueness of solutions is established and solutions are shown to be uniformly bounded for all nonnegative initial conditions. The model predicts that a deadly disease and a predator population cannot co-exist. Numerical simulations illustrate a variety of dynamical behaviors that can be obtained by varying the problem data.

21. *Computations of the Partition Function, p(n),* Elizabeth Perez, Wake Forest University and Jimena Davis, Clemson University, Clemson University REU

The partition function has been a subject of great interest for many number theorists over the past several years. Although it is based on fairly simple ideas on the surface, there are still many elementary questions unanswered. The work of the Indian mathematician, Srinivasa Ramanujan
(c.1887), has laid a basis for those interested in the theory of partitions. In 1966, two computational number theorists, Thomas Parkin and Daniel Shanks, wrote a paper on the parity of the partition function and supported their results with extensive computation, using the recursive definition for the partition function. Since then, an in-depth numerical analysis of partitions has not provided evidence to support the many conjectures being\ made in this area of mathematics. Today, especially in the past few years, great strides have been made and new ideas have taken shape by Ken Ono, Scott Ahlgren, and other leaders in the field. My poster outlines the history of computational work concerning the partition function. Furthermore, I will show some key points concerning the efficient algorithm we used for inverting a power series and computing the partition function modulo a large prime.

22. *Analyzing the Contractions of* Vorticella sp., Karoline Pershell, University of Tennessee-Martin and Florida State University REU

Spontaneous contractions of the stalk and spasmoneme of *Vorticella sp.* were observed using high speed video cinematography. Images were recorded at approximately 13,000 frames per second. Complete spasmoneme contraction duration averaged 53 ms. An average maximum contraction velocity of 2 cm/s was reached approximately 6 ms after the start. The data was collected from 18 specimens which represents an increased sampling volume compared to previous measurements collected by other research groups. A Lorentzian curve is suggested to explain the behavior of *Vorticella sp.* Close agreement of the collected data with the proposed model was found.

23. *2002 Penn State Erie REU in Mathematical Biology*, Meghan O'Malley, North Carolina State University and Penn State Erie REU

The 2002 Penn State REU in Mathematical Biology was designed to give the students an idea of the various topics and

methods in Mathematical Biology. The REU took place over the course of 5 weeks. After a week of introductory material, the students listened to lectures and worked on problems for different areas of Mathematical Biology for each of the following 3 weeks (Dynamics of Bursting in Neurons and Beta Cells, Pattern Formation in Coupled Neurons, Symbolic Dynamics and Plant Morphogenesis). During the last week, the students did further research on a topic of their choice.

24. *Infinite Dimensional Lagrangian Reduction*, Luke Cherveny, NC State University and REU at Trinity University, San Antonio

In this paper we attempt to apply the methods of Lagrangian reduction to problems in physical field theories by reducing physical field theories in a manner similar to established Euler-Poincar\'e reduction for particles or rigid body systems containing symmetry. Before starting, we survey Lagrangian reduction techniques and review principles from symplectic geometry, Lie group theory, and Riemannian geometry. We then reduce the electrodynamics problem by using the field tensor as a reduced variable and examine the similarities and differences in geometry with finite dimensional Lagrangian reduction. Finally, we then examine the Yang-Mills generalization of electrodynamics to model the weak nuclear force, attempt to perform a reduction, and ultimately learn that the non-Abelianness of SU(2) disallows the use of the field tensor as a reduced variable.

1:00-2:00 General Session IV

Ron Graham
University of California San Diego
Guessing Secrets

We will describe a variant of the familiar "20 Questions" problem in which one tries to discover the identity of some unknown
"secret" by asking binary questions. In this variation, there is now a set of two (or more) secrets to use in supplying the answers, which in any case must always be truthful. We will discuss a number of algorithms for dealing with this problem, although we are still far from a complete understanding of the situation. Problems of this type have recently arisen in connection with certain Internet traffic routing applications, although it turns out that such problems have in fact occurred in the literature more than 40 years ago.

2:20-3:20 Concurrent Sessions

Geometry

1. *Sums of Squares and Cubes: Proofs Without Many Words*, Stephen Curry , Georgia College and State University

Formulas for the sum of squares and the sum of cubes are demonstrated geometrically, without many words.

2. *Tangent sweeps and tangent clusters on the sphere and in the hyperbolic plane*, Irl Bivens, Davidson College

Suppose a line segment L moves along a smooth plane curve in such a way that one end of L is fixed to the curve and such that L is always tangent to the curve. The region swept out by the moving segment L is called the "tangent sweep" of L and the portion of a disk generated by collecting all the positions of L at a single point is called the "tangent cluster" of L. Recent articles (e.g. "Tangents and Subtangents Used to Calculate Areas" by Apostol and Mnatsakanian, MONTHLY, Dec., 2002, 900-907) point out that the area of the tangent sweep is equal to the area of the tangent cluster. In this talk we will show that analogous results also hold on the sphere and in the hyperbolic plane.

3. *A Generalization of Kasner's Theorem*, John Zerger, Catawba College

Consider the problem: Given a polygon P' in the plane which consists of vertices that are the midpoints of a polygon P, can
the original polygon P be constructed from P' ? Kasner solved this problem; showing that it depends on the parity of the number of edge on P. We will look at Kasner's theorem from both an algebraic and geometrical approach. In addition, we will generalize Kasner's Theorem by considering polygons, P', that are not necessarily constructed from midpoints but rather from other fractional lengths of the sides of the original polygon. Of course, in all cases we will not only answer the question "Can P be constructed?" but also show how to construct P, when possible.

Graph Theory

1. *Decompositions of the Complete Digraph into Orientations of Cycles*, Gary Coker, Francis Marion University

Let g be a subgraph of G. A *decomposition of G into copies of g* is a collection of isomorphic copies of g, $\{g_1, g_2, ..., g_n\}$ such that the union of the g_i's = G, with decompositions of digraphs similarly defined. An *n-cycle system* is a decomposition of the complete digraph into copies of an n-cycle. An analogous decomposition of digraphs is a decomposition of the complete digraph into copies of a digraph d, where d is some given orientation of a cycle. We explore such decompositions by presenting direct constructions in certain cases.

2. *Hamiltonicity of 2-Connected Quasi-Claw-Free Graphs*, Rao Li, University of South Carolina Aiken

A graph G is called quasi-claw-free if it satisfies the property: d(x, y) = 2 =>there exists a vertex u ∈ N(x) ∩ N(y) such that $N[u] \subseteq N[x] \cup N[y]$. Let G be a 2-connected quasi-claw-free graph of order n. If $\delta(G) \geq n/4$, then G is Hamiltonian or G ∈ ***F***, where ***F*** is a family of non-Hamiltonian graphs of connectivity 2.

Applied Mathematics

1. *The Greens Function Alternative in Industrial and Applied Mathematics*, Pascal Roubides, Georgia Tech

The article proposed presents an introductory discussion on an alternative technique for the solution of a large class of ordinary and partial differential equations. It is intended as an overview of the topic of Green's functions for advanced undergraduate students or graduate students in the field of applied mathematical physics and the various engineering disciplines. The discussion consists of the definition of the Greens function for linear boundary value problems, a proof of its existence and uniqueness based on the definition, examples of constructing Greens functions for sample boundary value problems, and a comparison of numerical approximations based on the Greens function against other traditional numerical methods, such as finite differences.

2. *A Maple Application of Splines and the function $x^p + y^p = 1$, $1<p<2$ in the Determination of the Quality of Coal*, Lyndell Kerley, East Tennessee State University

By using mixtures of chemicals in a lab with different specific gravities, a table of values for variables describing the ash and sulfur content as well as the BTU is generated. Unfortunately, for many of the specific gravities, the values of the variables are not determined by a lab. This is due to cost and the inability to obtain data at higher specific gravities due to the toxic natures of such chemical with such high specific gravities. Thus an interpolation problem is presented. How does one use interpolation to approximate the variables at say 24 specific gravities ranging from 1.24 to 2.7 where only data is provided at say 1.40,1.45,1.5,1.6,1.7? Spline interpolation provides an approximation up to say 1.7. The curve $x^p + y^p = 1$, $1<p<2$ provides an approximation from 1.7 to 2.1. What is p? An algorithm will be presented for determining the appropriate p. For p = 1, $x^p + y^p = 1$ is a line whereas for p=2, we have a circle. In our application, we will see a satisfactory solution is found for our given data set where p = 1.56. Maple will be used to obtain the solution.

3. *Pricing American Options via Monte Carlo: A Variance Reduction Technique*, Tracey Tullie, North Carolina Agricultural and Technical State University

In recent years, there has been much attention given to the area of financial mathematics. Companies are utilizing tools within financial mathematics to find optimal ways in which to manage their risk. Options are one tool that companies utilize to help manage risk. In this paper, we propose a variance reduction technique for Monte Carlo simulation of American options in the context of stochastic volatility. The method is based upon importance sampling using an initial estimate of the option price. The approximation is obtained by one of two forms of expansion, regular and singular perturbation, of the pricing partial differential inequality. We demonstrate numerically the efficiency of utilizing either form of expansion for an initial approximation of the price.

Miscellaneous I

1. *Crash Course in Context-Oriented Mathematical Logic*, Damon Scott, Francis Marion University\

In his book, *The Situation in Logic*, the late Jon Barwise sought to discover a system of context-oriented mathematical logic. By his own admission, he sought but he did not find. In this lecture, we present a surprisingly simple system that does the job. Examples will be given of how mathematical situations are aptly formalized with the new calculus. Like speaking in prose, mathematicians will find that they have been doing their mathematics in context-oriented fashion for years. Well-formed mathematical contexts will be defined, their logical structure built up, and the usual sufficiency theorems will be stated, though without proof. The new system is not just a change of names from traditional syntax but a change of *phrase structure*, and it is thereby that mathematics looks so very different when expressed in context-oriented form. We intend to present, as time permits, the most striking of these changes: how the amount of nestedness of expression is vastly reduced from traditional, statement-oriented mathematical logic, how consequently mathematical work in practice falls into only a handful of *scales of expression*, and how the new system is remarkably close to the forms of formal proof. Quite simply, the new system is much more user-friendly than the traditional statement-oriented system and it ‘‘thinks’’ much more in the same way that mathematicians do in their practice of the discipline.

2. *Inverse Iteration of Elliptic Functions*, Mark McClure, University of North Carolina at Asheville

Inverse iteration is a technique for vizualizing the Julia set of a function mapping the complex plane to itself. Elliptic functions are

functions which are doubly periodic and meromorphic on the complex plane and have a long history in the study of complex dynamics. In this talk, we present some images generated by inverse iteration of Elliptic functions and discuss some of the difficulties in its implementation

3. *Weighted Weak Type Inequalities for Hardy Operator When p = 1*, Tieling Chen, University of South Carolina Aiken

This paper studies the weighted weak type inequalities for the Hardy operator as an operator from the weighted

Lebesgue L^pspace to the weighted Lebesgue L^q space in the case p = 1. It proves that the weak type inequality and the strong type inequality are equivalent when p = 1, which is not generally true in other cases. It also gives a necessary and sufficient condition for the weighted weak type inequality for the classical Hardy operator when p = 1. The best constant in the inequality is estimated.

Miscellaneous II

1. *Paper Folding and an Angle Limit: A Surprising Result,* Scotty Fairbairn, Clemson University

Start with a strip of paper and fold over an angle of any measure. Via folding, halves of supplements of preceding angles are produced. The sequence of angle measures approaches a constant. The proof of this surprising result involves sequences, series and limits and is pretty.

2. *Hesiod's Falling Anvil,* Andrew Simoson, King College

The Greek poet Hesiod made the claim in about 700 BC that an anvil dropped from the Earth's surface would reach the Earth's center in 9 days, and that an anvil dropped from Heaven would reach the Earth in 9 days as well. With respect to an extrapolation of the physics for that day up through the days of Galileo, we examine whether these figures are at all reasonable, and then compare them with corresponding figures from the physics of today.

3. *Light Beam Switching at the Interface of Two Nonlinear Optical Media,* Rajah P. Varatharajah, North Carolina A&T State University

The dynamics of two light beams interacting at an interface separating two nonlinear optical media is studied numerically and analytically. The beam dynamics is simulated by the beam propagation method. The numerical simulations agree with the results of analytical model. The trapped beam at the interface acts as a power controllable switch to reflect or transmit the incident beam at the interface. These results can be used to develop a device in signal switching.

Special Session on Discrete Mathematics III

SATURDAY, March 22, 2:20 - 2:45

Special Session on the History of Mathematics III

1. *Comparing the van Hiele Model to the Piaget Model*, Rachel Keller, Clemson University

According to the van Hiele Model of the Development of Geometric Thought, a student of geometry passes through 5 stages of understanding. These labels- visualization, analysis, informal deduction, formal deduction, and rigor- represent a student's level of geometric maturity and dictate what types of problems he/she will be able to solve at each level. The purpose of this talk is to compare these van Hiele levels with the Stages of Cognitive Development as identified by Jean Piaget. These stages- sensorimotor, pre-operational, concrete operational, and formal operational- follow the progression in cognitive development from infancy through adulthood. Since Piaget's levels govern general thought processes and are not specific to mathematics, it will be interesting to see if the van Hiele levels can be predicted from the characteristics Piaget claims we should expect with certain levels of cognitive development.

2. *Reflections on Zeno's Paradoxes*, Dan Sloughter, Furman University

In the fifth century B.C., Zeno of Elea wrote a book in defense of the teachings of his mentor, Parmenides. The book itself is long since lost, but four of Zeno's arguments against motion were preserved in the writings of Aristotle. We will discuss interpretations of Zeno's paradoxes and their influence on subsequent thought, with particular attention to the writings of C. S. Peirce.