## Homework 2, due: 02/03

MATH 9830, Spring 2015
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0 . Before you start:

- Get deal.II to work on your computer.
- Read the description of tutorial step-1 and step-2, optionally watch the linked videos.
- Make sure you can compile and run both tutorials.

1. Edit deal.II step-1 to create an image of an L-shape domain that is adaptively refined a few times around the re-entrant corner.
2. Edit deal.II step-2 to print out the half-bandwidth $p$ after Cuthill-McKee for a hyper_cube with $1,2, \ldots, 9$ global refinements in 2d. Plot the bandwidth over the size of the matrix in a $\log -\log$ plot.
Estimate a relationship between $p$ and the number of unknowns $n$. What complexity would the LU decomposition have if we assume $O\left(p^{2} n\right)$ and why is that a pessimistic assumption?
3. Consider a sparse matrix $A$ with the following pattern:

$$
\left(\begin{array}{lllllll}
\star & \star & & & & & \\
\star & \star & \star & & & \star & \\
\\
& \star & \star & \star & & & \\
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& & \star & & & \star & \star \\
& \star \\
& & & \star & & & \star \\
& \star
\end{array}\right) .
$$

(a) Show the adjacency graph of $A$ (place the 8 vertices on a circle).
(b) Now apply a reverse Cuthill-McKee ordering starting at the first node (add the level number to your graph in a)). Keep an increasing order for the vertices with the same level (so vertex 7 will be the first one in your permutation). Draw the new sparsity pattern.
(c) Pretend you are computing an LU decomposition, once with the original matrix and once with the permuted matrix. Mark every new non-zero entry with a different symbol (maybe use ' $x$ ' and 'o'). Count the number of new entries and compare.

