Limit Laws:

$\mathcal{S}$ $c \in \mathbb{R}$ and $\lim_{x\to a} f(x)$ & $\lim_{x\to a} g(x)$ both exist. Then

1. $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

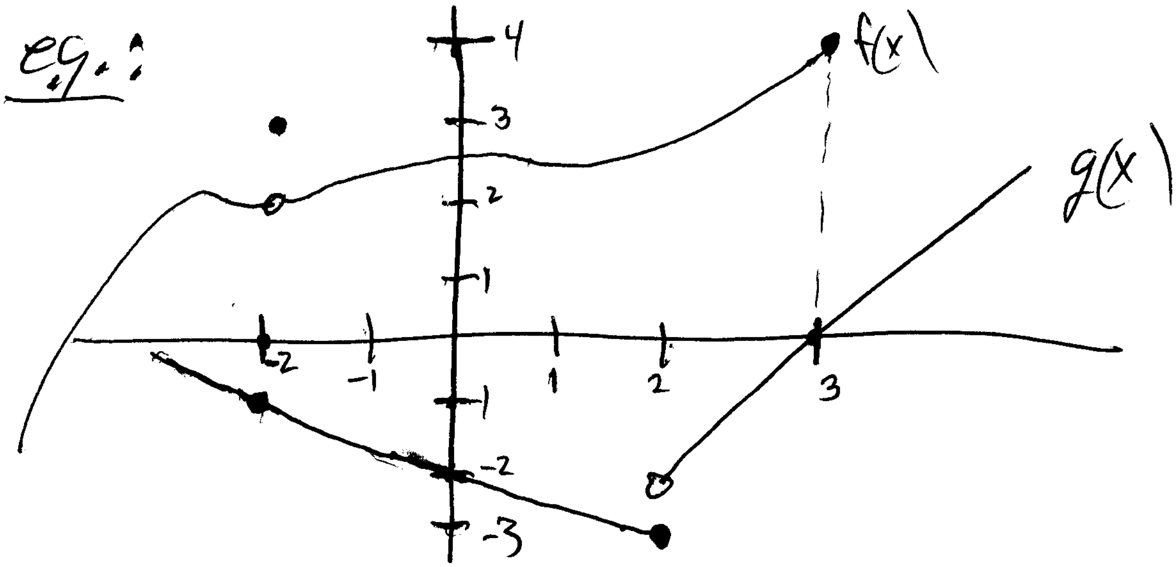
2. $\lim_{x\to a} [f(x) - g(x)] =$

3. $\lim_{x\to a} (c\, f(x)) =$

4. $\lim [f(x)\, g(x)]$

5. $\lim_{x\to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ provided $\lim_{x\to a} g(x) \neq 0$.

eg.:



$$\lim_{x\to 3} (f+5g) = ?$$

$$\lim_{x\to 2} fg = ?$$

$$\lim_{x\to -2} \left(\frac{f}{g}\right) = ?$$

More limit laws:

6.) $\lim_{x \to a} [(f(x))^n] = [\lim_{x \to a} f(x)]^n$ (n is a positive integer)

7.) $\lim_{x \to a} c = c$

8 $\lim_{x \to a} x = a$

9.) $\lim_{x \to a} x^n = [\lim_{x \to a} x]^n = a^n$

where n is a positive integer.

10.) $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$

(where n is a positive integer. If n, even, $a > 0$.)

11.) $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$

where n is a positive integer.

(If n, even $\lim_{x \to a} f(x)$ must be > 0.)

e.g.

- $\lim_{x \to 2} 2x^2 + 3x + 5 = ?$
- $\lim_{x \to 1} \left[\dfrac{x^3 + 3x^2 + 2x + 6}{5x + 3} \right] = ?$

Chalk board. Quickly!

- Fact: If f is a poly. ~~[illegible]~~ or a rational function (i.e. ratio of poly.'s) and a is in the domain of f (i.e. the bottom is not zero) then

$$\lim_{x \to a} f(x) = f(a).$$

Note: Such fc.'s are called cont.

e.g. $\lim_{x \to 1} \dfrac{x^2 - 1}{x - 1}$

Fact: If $f(x) = g(x)$ when $x \neq a$ then $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$.

e.g.:
$$g(x) = \begin{cases} x+1 & \text{if } x \neq 1 \\ \pi & \text{if } x = 1 \end{cases}$$

$$\lim_{x\to 1} g(x) = \lim_{x \to 1} x+1 = 2.$$

Maybe skip.

e.g.:
$$\lim_{t\to 0}\left[\frac{\sqrt{t^2+9}-3}{t^2}\right] = \lim_{t\to 0}\left[\frac{\sqrt{t^2+9}-3}{t^2}\cdot\frac{(\sqrt{t^2+9}+3)}{(\sqrt{t^2+9}+3)}\right]$$

$$= \lim_{t\to 0}\left[\frac{t^2+\cancel{9}-\cancel{9}}{t^2(\sqrt{t^2+9}+3)}\right] = \lim_{t\to 0}\left[\frac{1}{\sqrt{t^2+9}+3}\right]$$

$$= \frac{1}{6}$$

Thm: $\lim_{x\to a} f(x) = L$ iff $\lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$

e.g.
$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$\lim_{x\to 0^-} |x| = ?$ $\qquad \lim_{x\to 0^+} |x| = ?$ $\qquad \lim_{x\to 0} |x| = ?$

$\lim_{x\to 0^-} \frac{|x|}{x} =$ $\qquad \lim_{x\to 0^+} \frac{|x|}{x} = ?$ $\qquad \lim_{x\to 0} \frac{|x|}{x}$

• See other examples in book !!!

- **Thm:** If $f(x) \le g(x)$ when x near a except at $x = a$ then

$$\lim_{x \to a} f(x) \le \lim_{x \to a} g(x).$$

- **Squeeze Thm:**

If $f(x) \le g(x) \le h(x)$ when x is near a except possibly at $x = a$, and

$$\lim_{x \to a} f(x) = L = \lim_{x \to a} g(x)$$

then

$$\lim_{x \to a} h(x) = L.$$

eg.:

$$\lim_{x \to 0} x^2 \sin \frac{1}{x} = ?$$

Note: $-x^2 \le x^2 \sin \frac{1}{x} \le x^2$

$$\lim_{x \to 0} -x^2 = 0 = \lim_{x \to 0} x^2$$

$$\therefore \lim_{x \to 0} \left[x^2 \sin \frac{1}{x}\right] = 0$$