- **Def:** A fc. $f(x)$ is continuous at a provided that

$$\lim_{x \to a} f(x) = f(a)$$

- To check continuity we must check 3 things:

1-> $f(a)$ is defined.

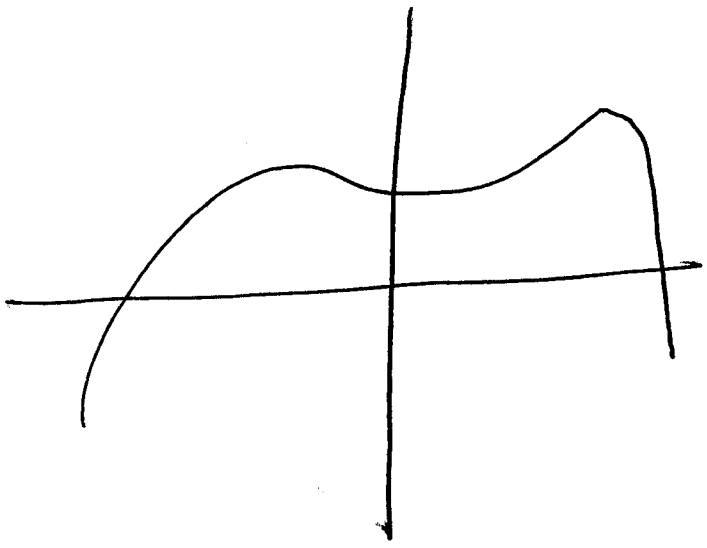
2.> $\lim_{x \to a} f(x)$ exists

3-> $\lim_{x \to a} f(x) = f(a)$

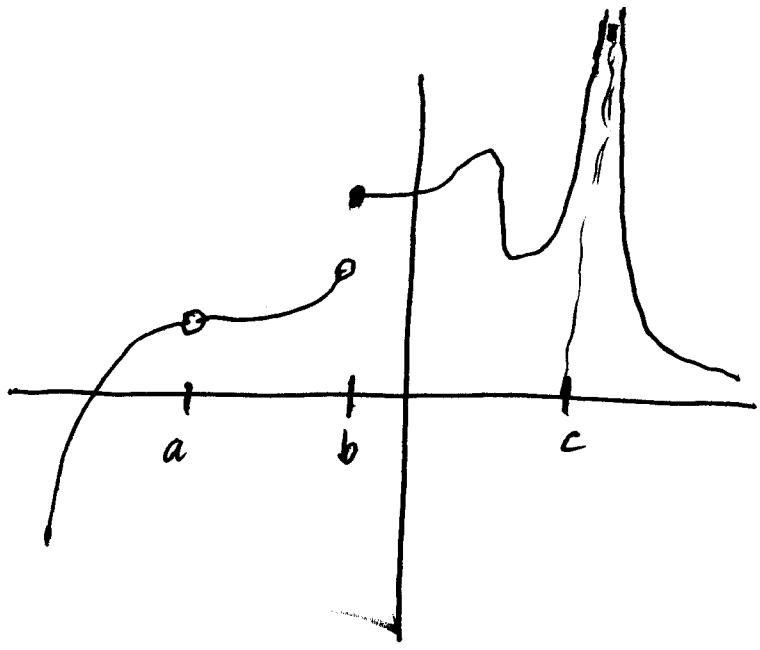
- **Def:** If f is not cont. at a we say that f is discontinuous at a or that f has a discontinuity at a.

eg.

continuous

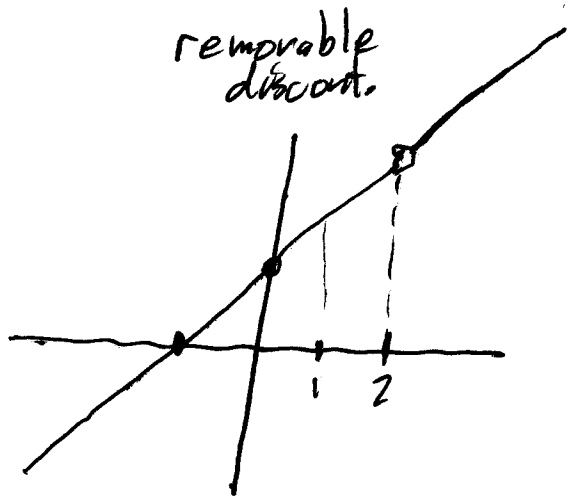


dis cont. at a, b & c

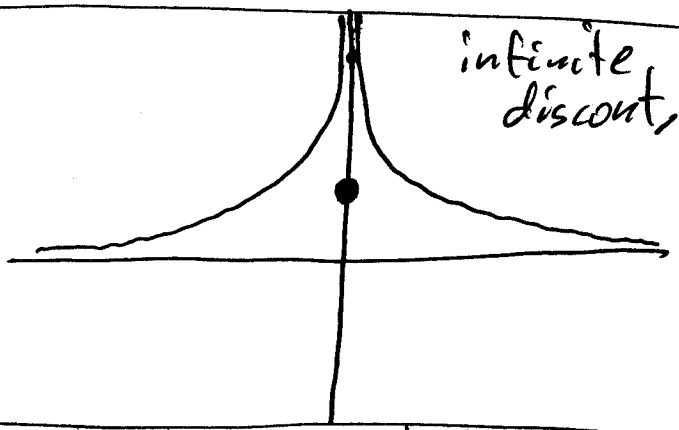


eg: Locate the discontinuities of

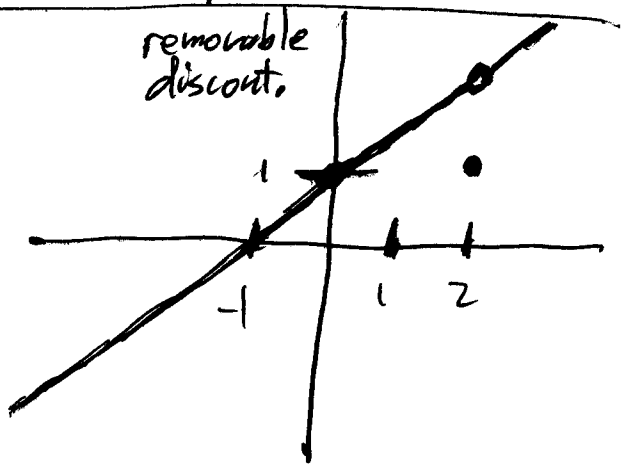
a) $f(x) = \dfrac{x^2 - x - 2}{x - 2}$



b) $f(x) = \begin{cases} \frac{1}{x^2} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$



c) $f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$

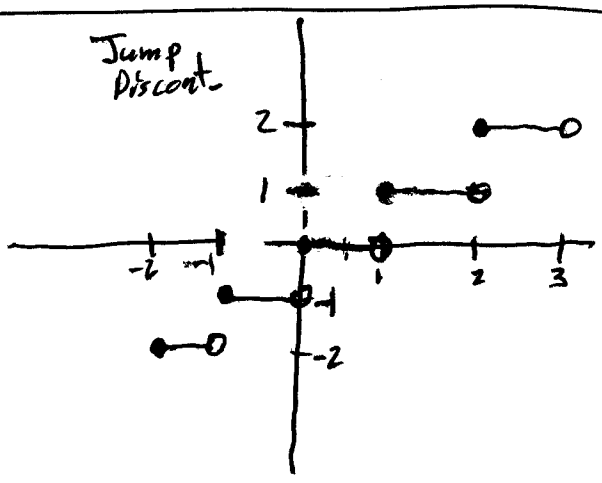


d) $f(x) = [\![x]\!]$

$[\![x]\!]$ is the greatest integer $\leq x$

$[\![2]\!] = 2$

$[\![1.75]\!] = 1$



- Def: A fc. is cont. from the left at a number a provided that

$$\lim_{x \to a^-} f(x) = f(a)$$

A fc. is cont. from the right provided that

$$\lim_{x \to a^+} f(x) = f(a)$$

- eg.: $[\![x]\!]$ is cont. from the right everywhere.

- Def: A fc f is cont. on an interval if it is cont. at each point in the interval.

(Cont. at the endpoints of an interval means cont. from the left or the right.)

eg. show that $f(x) = 1 - \sqrt{1-x^2}$ is cont. on $[-1, 1]$

• **Fact:** If $f(x)$ & $g(x)$ are cont. at a and $c \in \mathbb{R}$ then the following functions are also cont. at a.

1.) $f(x) + g(x)$

2.) $f(x) - g(x)$

3.) $c\,f(x)$

4.) $f(x)\,g(x)$

5.) $f(x)/g(x)$ provided $g(a) \neq 0$.

• **Fact:**

1.) Any polynomial is cont. on $(-\infty, \infty)$

2.) Any rational fc. is cont. on its domain.

eg. Let $f(x) = \dfrac{x^2+1}{5-3x}$.

Then f is cont. on $(-\infty, 5/3) \cup (5/3, \infty)$

Thus, $\lim_{x \to 5} f(x) = f(5) = \dfrac{26}{-10} = -2.6$

Fact: The following fc.'s are cont. everywhere in their domains,

polynomials rational fc.'s

root fc.'s trig. fc.'s

eg. On what interval is $f(x) = \sqrt{x} + \frac{x+1}{x-1} - \frac{x+1}{x^2+1}$ continuous?

Fact: (1.) If f is cont. at b and

$$\lim_{x \to a} g(x) = b$$

then

$$\lim_{x \to a} f(g(x)) = f(b)$$

(i.e. $\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$)

(2.) If g is cont. at a and f is cont. at $g(a)$ then $f \circ g$ is cont. at a.

• e.g. Where is

$$F(x) = \frac{1}{\sqrt{x^2+7} - 4}$$

cont.?

• Thm. (Intermediate Value Theorem).

Suppose that f is cont. on $[a,b]$ and let N be a number between $f(a)$ and $f(b)$. (i.e. $f(a) < N < f(b)$ or $f(b) < N < f(a)$). Then there is a number $c \in (a,b)$ such that $f(c) = N$.

• e.g. show that there is a root to the eq.

$$4x^3 - 6x^2 + 3x - 2 = 0$$

in $[1,2]$.