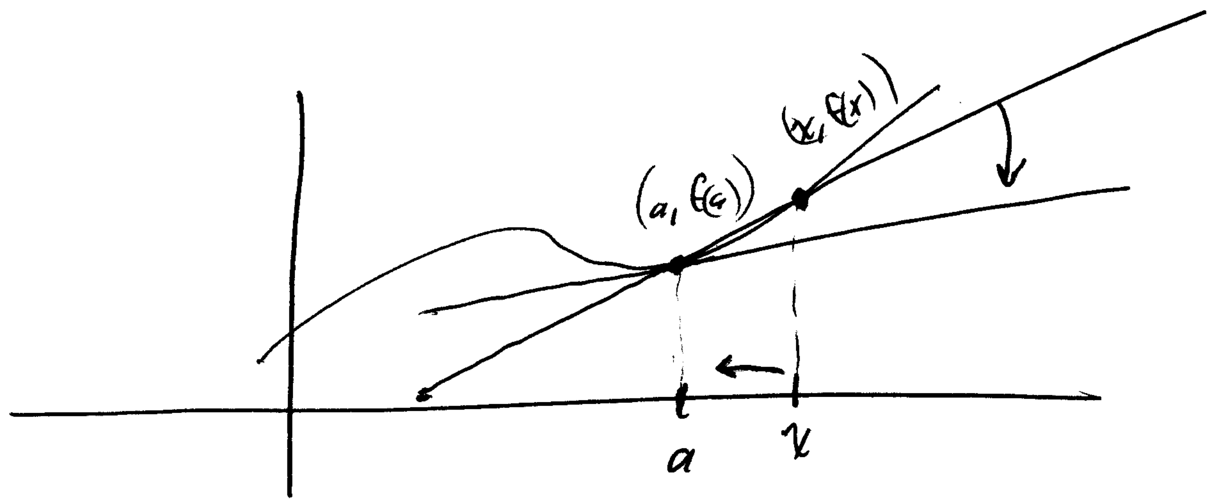
2.6 Tangents Velocities & other Rates of change

Def:

The tangent line to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope:

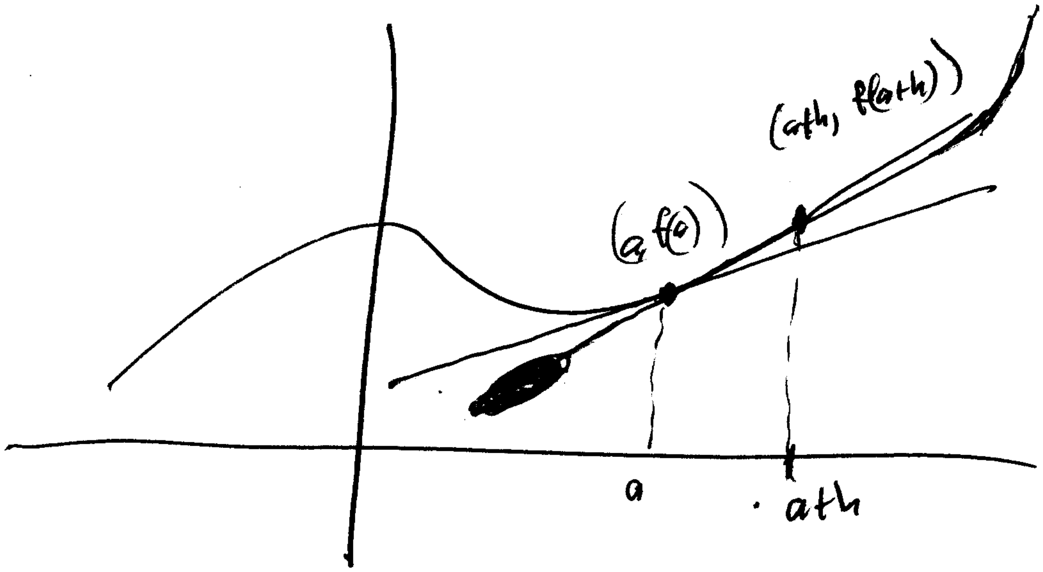
$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided the limit exists.



eg: Find the eq. of the tangent line to the parabola $y = x^2$ at $P(1,1)$.

Alternate Def:



Letting $h \to 0$ sends $a+h$ to a,
so, we define the slope of the tangent line as

$$\boxed{m = \lim_{h \to 0} \left[\frac{f(a+h) - f(a)}{h} \right]}$$

e.g.: Find the eq. of the tangent line to $y = \frac{10}{x}$ at $(1, 10)$

eg. Find the slope of the tangent line to $y = \sqrt{x}$ at any pt. $(a, \sqrt{a})$.

· **Velocity:**

~~Def:~~ If the position of an object at [illegible] time t is given by $f(t)$, then the instantaneous velocity at time $t = a$ is

$$\boxed{v(a) = \lim_{h \to 0} \left[\frac{f(a+h) - f(a)}{h} \right]}$$

eg.: If a ball is dropped from 450m above the ground.

a) what is the velocity after 5s

b) How fast is the ball moving when it hits the ground.

Position fc.: $4.9t^2$

Other Rates of change.

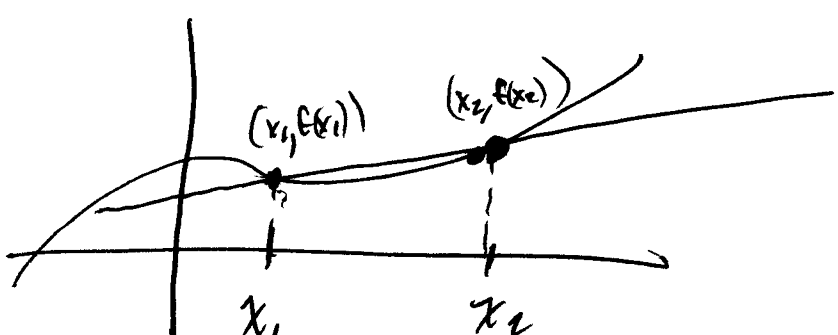
Suppose y is a function of x, say
$$y = f(x).$$

If x changes from x_1 to x_2 then the increment of x is
$$\Delta x = x_2 - x_1$$

The corresponding change in y is:
$$\Delta y = f(x_2) - f(x_1)$$

The average rate of change of y w.r.t. x is:
$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Note: This is also the slope of the secant line through $(x_1, f(x_1))$ $(x_2, f(x_2))$.



The instantaneous rate of change of y w.r.t. x at $x = x_1$ is:

$$\lim_{\Delta x \to 0} \left[\frac{\Delta y}{\Delta x} \right] = \lim_{x_2 \to x_1} \left(\frac{f(x_2) - f(x_1)}{x_2 - x_1} \right)$$

$$\left(= \lim_{h \to 0} \left(\frac{f(x_1 + h) - f(x_1)}{h} \right), \right)$$