- <u>Recall</u>: Slope of the tangent line to $y = f(x)$ at $(a, f(a))$ is

$$m = \lim_{h \to 0} \left[\frac{f(a+h) - f(a)}{h} \right]$$

- <u>Def</u>: The derivative of a fc. f at a number a is defined as

$$f'(a) = \lim_{h \to 0} \left[\frac{f(a+h) - f(a)}{h} \right]$$

<u>OR</u>

$$f'(a) = \lim_{x \to a} \left[\frac{f(x) - f(a)}{x - a} \right]$$

- <u>eg.</u> Let $f(x) = x^2 + 3x + 4$.
Compute $f'(a)$.

Tangent Line:

The tangent line to $y = f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ with slope $f'(a)$

It has equation

$$y - f(a) = f'(a)(x-a)$$

eg. Find the equation of the line tangent to $y = x^2 + 2x + 1$ at $(1, 4)$.

Rates of Change.

If $y = f(x)$, recall

if $\Delta x = x_2 - x_1$

$\Delta y = f(x_2) - f(x_1)$

and the instantaneous rate of change is

$$\lim_{x_2 \to x_1} \left(\frac{\Delta y}{\Delta x} \right) = \lim_{x_2 \to x_1} \left[\frac{f(x_2) - f(x_1)}{x_2 - x_1} \right]$$

$$= f'(x_1)$$

Fact: The derivative $f'(a)$ of f at a is the instantaneous rate of change of $y = f(x)$ w.r.t. x when $x = a$.

(See example 5 pp. 130–131.)

eg. The position of a particle is given by

$s(t) = \frac{1}{1+t}$ (t is in seconds; position is in m)

Find the velocity of the particle at time $t = 2s$. What is the speed?