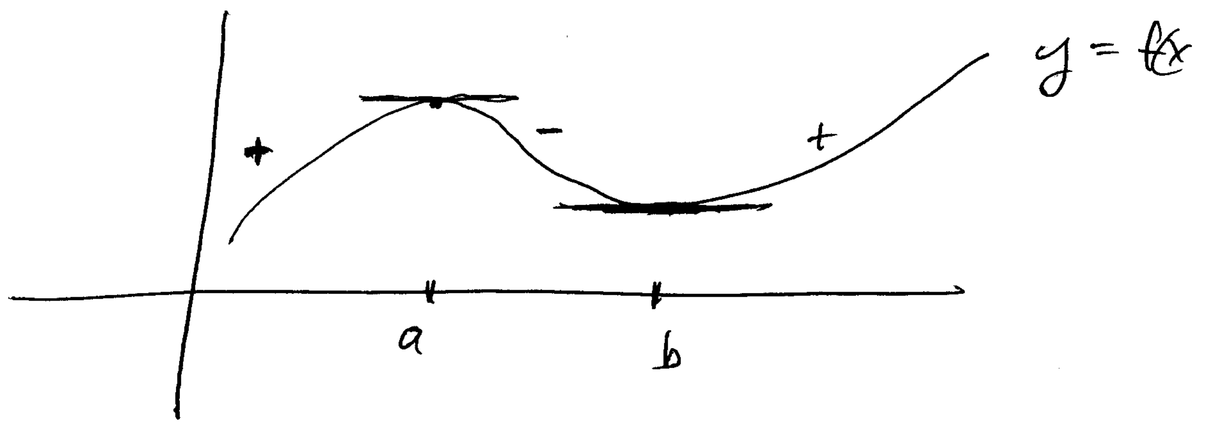
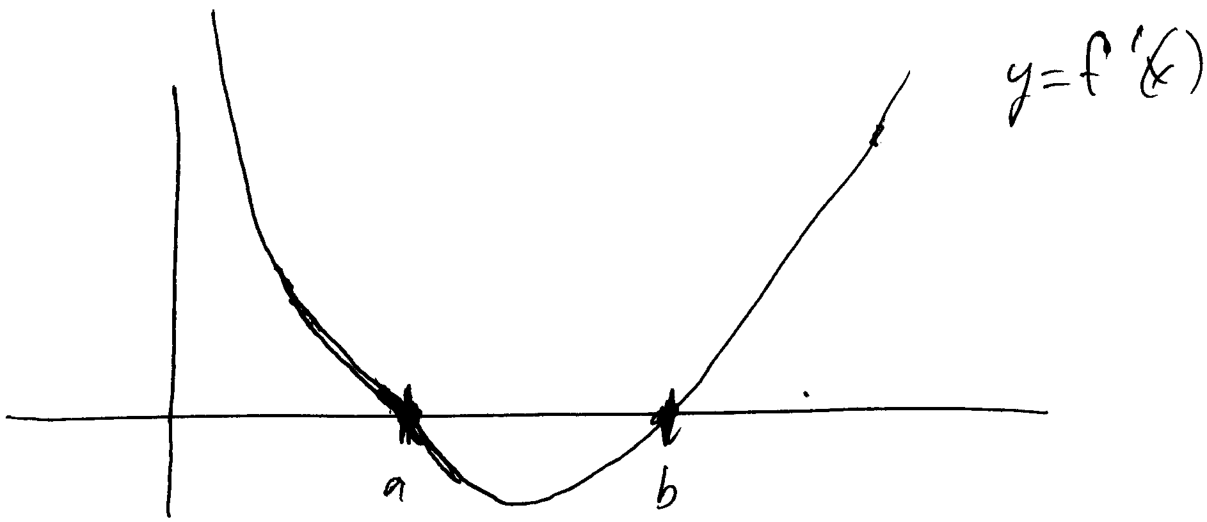
Def:

$$f'(x) = \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right]$$

eg.:





e.g.: Let $f(x) = x^3 + 4x + 2$
Find a formula for $f'(x)$.

e.g.: Let $f(x) = \sqrt{x-1}$
Find the derivative and state its domain.

• **Notation:** If $y = f(x)$ we might denote the derivative as

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x) = D f(x)$$

$$= D_x f(x) .$$

Def: A fc. f is differentiable at a iff $f'(a)$ exists. It is diff. on (a,b) if it is diff. at every number in (a,b).

eg.: Where is $f(x) = |x|$ diff.?

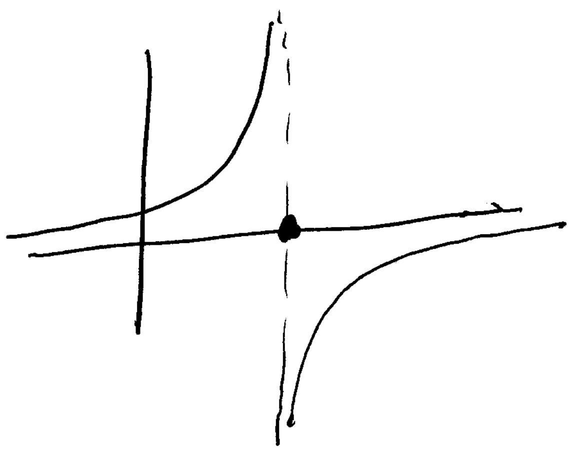
Recall $|x| = \begin{cases} -x & \text{if } x<0 \\ x & \text{if } x \geq 0 \end{cases}$

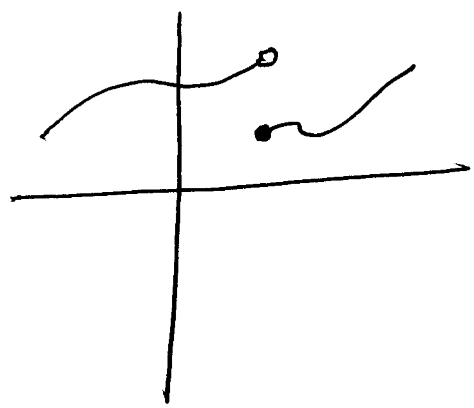
- Thm: If f is differentiable at a then it is continuous at a.

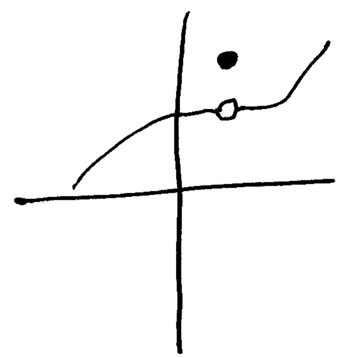
- Note: Cont. does not imply diff.
  (eg. $f(x)=|x|$ is cont. but not diff. at 0.)

- How can a function fail to be diff.?

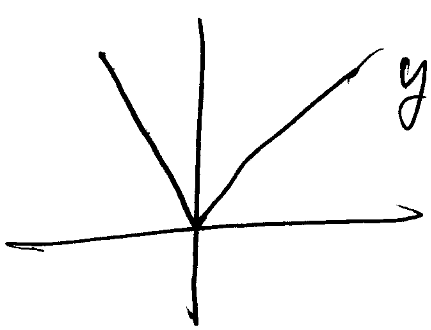
1.) It could fail to be cont.

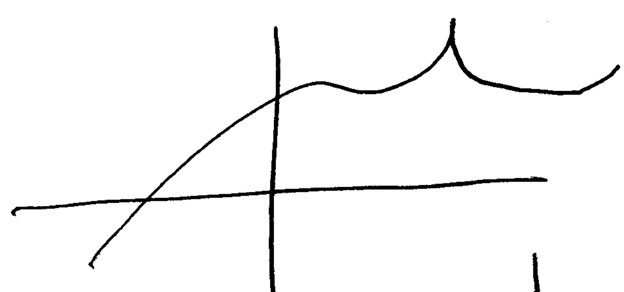






2.) It could have a sharp point.





3.) It could have a vertical tangent line.
(eg. graph $y=x^{1/3}$)