§3.3 Diff. Formulas :

1) $\frac{d}{dx} c = 0$

2) $\frac{d}{dx} x = 1$

3) $\frac{d}{dx} x^n = nx^{n-1}$

Pf: $f(x) = x^n$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{x^n - a^n}{x-a}$$

$$= \lim_{x \to a} \frac{(x-a)(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \cdots + xa^{n-2} + a^{n-1})}{(x-a)}$$

$$= \lim_{x \to a} \left(x^{n-1} + x^{n-2}a + \cdots + a^{n-1} \right)$$

$$= \underbrace{x^{n-1} + x^{n-1} + \cdots + x^{n-1}}_{n \text{ terms}} = nx^{n-1}.$$

e.g. $f(x) = x^4$ $\qquad$ $g(x) = x^{12}$

$f'(x) =$ $\qquad$ $g'(x) =$

4) $\frac{d}{dx}\, c\, f(x) = c\, \frac{d}{dx} f(x)$

e.g. $f(x) = 5x^4$

$f'(x) =$

5) $\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

e.g. $f(x) = x^3 + 3x^2 + 2x + 5$

$f'(x) =$

6) Product Rule:

$$\frac{d}{dx}\left(f(x)g(x)\right) = \frac{d}{dx} f(x)\; g(x) + f(x)\, \frac{d}{dx} g(x)$$

<u>eg.</u> $f(x) = (x^2+2)(x^{10}+3x+4)$

$f'(x) =$

• <u>Quotient Rule:</u>

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)\,g(x) - g'(x)\,f(x)}{g(x)^2}$$

<u>eg.</u> $f(x) = \dfrac{x^3+3x^2+3x+1}{x^2+5x+6}$

$f'(x) =$

eg.: Compute $\frac{d}{dx}\left[x^{\frac{1}{2}}\right]$

$$\frac{d}{dx}\left[a^{\frac{1}{2}}\right] = \lim_{h\to 0} \frac{\sqrt{a+h}-\sqrt{a}}{h} \cdot \frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}}$$

$$= \lim_{h\to 0}\left[\frac{(a+h)-a}{h(\sqrt{a+h}+\sqrt{a})}\right] = \lim_{h\to 0}\left[\frac{\cancel{h}\ 1}{\cancel{h}(\sqrt{a+h}+\sqrt{a})}\right]$$

$$= \frac{1}{2\sqrt{a}} = \frac{1}{2}a^{-\frac{1}{2}}.$$

Thus, $\frac{d}{dx}\left(x^{\frac{1}{2}}\right) = \frac{1}{2}x^{-\frac{1}{2}}$.

. General Power Rule:

$$\frac{d}{dx}\left(x^n\right) = n x^{n-1} \quad \forall n \in \mathbb{R}.$$

eg. $f(x) = \dfrac{\sqrt{x}-1}{\sqrt{x}+1}$

$f'(x) =$