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§3.4:

Recall: If x changes from x_1 to x_2 and if $y = f(x)$ then we write

$\Delta x = x_2 - x_1$ (the change in x)

$\Delta y = f(x_2) - f(x_1)$ (the change in y)

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$ (the average rate of change of y w.r.t. x in the interval $[x_1, x_2]$)

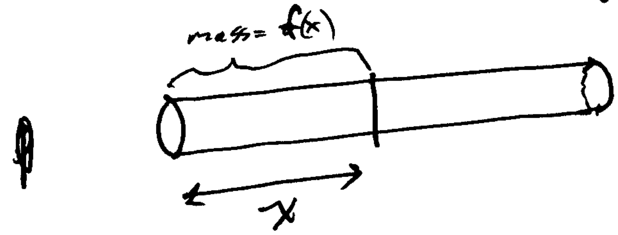
$$\frac{dy}{dx} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$ (the instantaneous rate of change in y w.r.t. x when $x = x_1$)

* **Homework**: Interview at least two people who are employed in your major field of study and write a paragraph on why you need to know calculus for that field.

eg. Physics :

Suppose that a rod is made of a non homogeneous mixture of materials and that the mass of the rod between its left end pt. and the pt. x meters to the right is given by

$$f(x) = \sqrt{x} \text{ kg.}$$



• Mass between x_1 & x_2 is :

$$dm = f(x_2) - f(x_1)$$

• Linear density at x_1 is :

$$\rho = \lim_{x_2 \to x_1} \left(\frac{f(x_2) - f(x_1)}{x_2 - x_1} \right)$$

$$= \frac{dm}{dx} \text{ OR } f'(x), \text{ kg/m}$$

eg. The linear density of the rod at $x = 1.1$ is

$$f'(1.1) = \frac{1}{2\sqrt{1.1}} \approx 0.4767 \text{ kg/m}$$

e.g. (Chemistry):

In a chemical reaction such as

$$2H_2 + O_2 = 2H_2O$$

we have two <u>reactants</u> reacting to form a <u>product</u>

$$\underbrace{A + B}_{\text{reactants}} = \underbrace{C}_{\text{product}}$$

We will denote the concentration in moles/liter of the reactants and product above at time t as

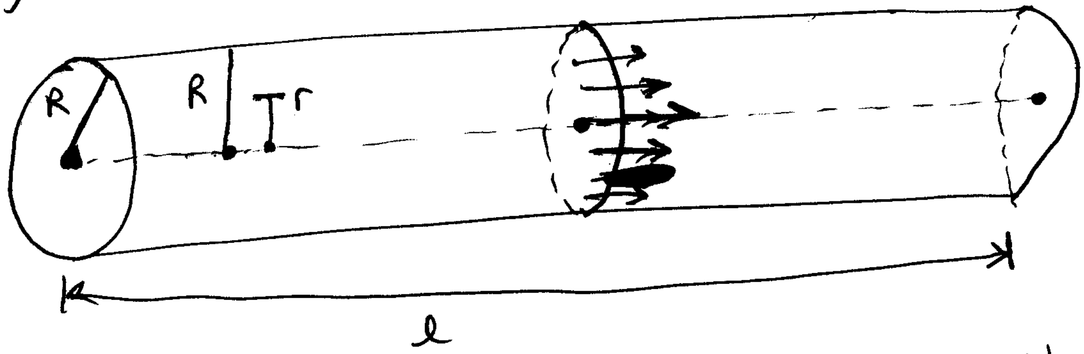
$$A(t),\ B(t),\ C(t)$$

- Avg. rate of reaction $= \dfrac{\Delta C}{\Delta t} = \dfrac{C(t_2) - C(t_1)}{t_2 - t_1}$
- Instantaneous rate of reaction $= \dfrac{dC}{dt} = \lim_{t_2 \to t_1} \left[\dfrac{C(t_2) - C(t_1)}{t_2 - t_1} \right]$

It turns out that A & B decrease at the same rate as C increases. That is,

$$\frac{dC}{dt} = -\frac{dA}{dt} = -\frac{dB}{dt}$$

– eg, Biology: Blood Vesselel,



The flow is fastest at the center of the vessel and slows as we move toward the vessel wall.

Let v denote the velocity of the flow. Then v decreases as r increases until $v=0$ when $r=R$.

- Law of Laminar flow:

$$v = \frac{P}{4\eta l}(R^2 - r^2)$$

P = diff of press on both ends

η = blood viscosity.

Note: If P and l are fixed then v is a fc. of r w/ domain $[0, R]$.

$$\text{avg. rate of change} = \frac{\Delta v}{\Delta r} = \frac{v(r_2) - v(r_1)}{r_2 - r_1}$$

$$\text{velocity gradient} = \lim_{r_2 \to r_1} \frac{v(r_2) - v(r_1)}{r_2 - r_1} = v'(r_1)$$

For small human arteries

$R = 0.008$ cm

$\eta = 0.027$

$l = 2$ cm

- How fast is the blood flowing when $r = 0.002$, $r = 0.004$.

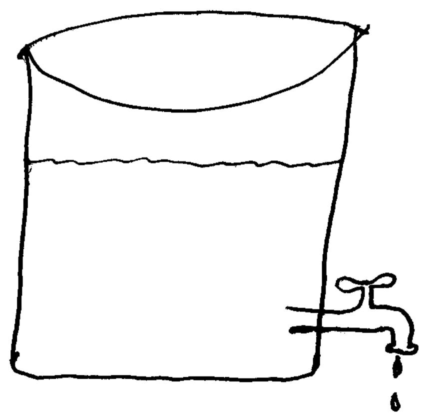
$v(0.002) =$

$v(0.004) =$

- What is the velocity gradient when $t = 0.004$?

$\frac{dv}{dt} = ?$

Torricelli's Law:



5000 gal tank

Water drains completely in 40 min.

$$V(t) = 5000\left(1-\frac{t}{40}\right)^2 \text{ gal/min}$$

for $t \in [0,40]$

velocity of water flow

- How fast is the water flowing at $t = 5, 10, 20, 30$ min

$v(5) =$

$v(10) =$

$v(20) =$

$v(30) =$

- when is it flowing fastest?

- what is the instantaneous rate of change of the flow at time t?

$$v(t) = 5000\left(1 - \frac{t}{20} + \frac{t^2}{1600}\right)$$

$$= 5000 - 250t + \frac{25}{8}t^2$$

$v'(t) =$