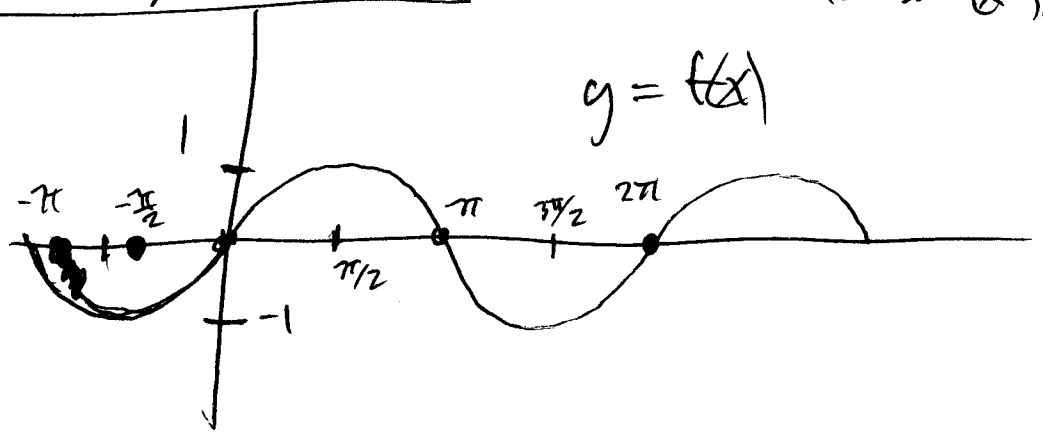
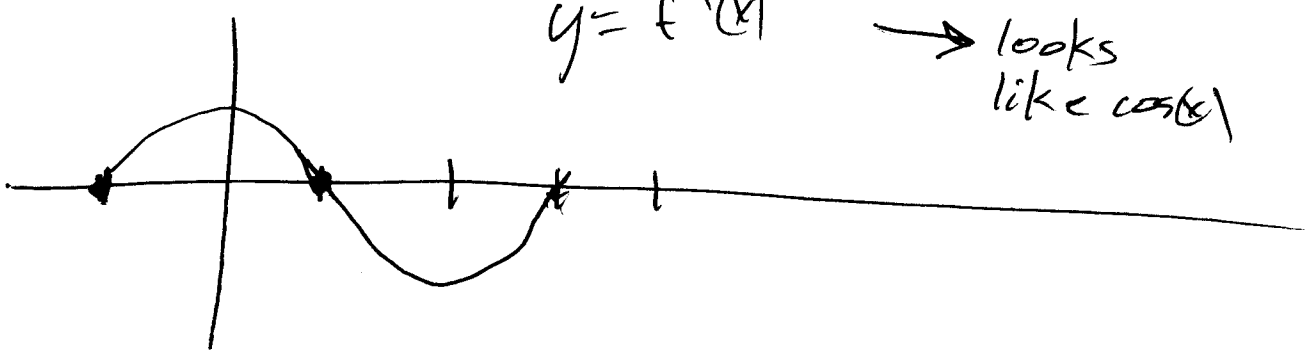
§3.5 Trig functions

Let $f(x) = \sin(x)$.





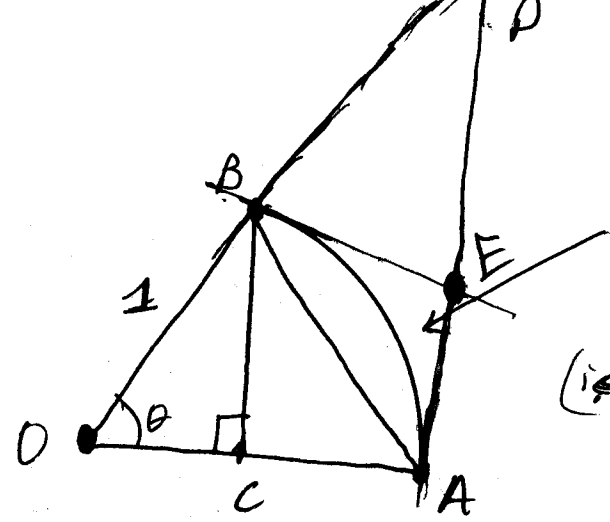
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(h)\cos(x) + \sin(x)\cos(h) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \left[\sin(x)\left(\frac{\cos h - 1}{h}\right) + \cos(x)\left(\frac{\sin(h)}{h}\right) \right]$$

$$= \lim_{h\to 0} (\sin(x)) \cdot \lim_{h\to 0}\left(\frac{\cos(h)-1}{h}\right) + \lim_{h\to 0} (\cos(x)) \cdot \lim_{h\to 0}\left(\frac{\sin(h)}{h}\right)$$

$$= \sin(x) \cdot \lim_{h\to 0}\left(\frac{\cos(h)-1}{h}\right) + \cos(x) \cdot \lim_{h\to 0}\left(\frac{\sin(h)}{h}\right)$$



circle of radius 1

(i.e. $|OA| = |OB| = 1$)

arc AB = radius $\cdot \theta = \theta$

$|BC| = \sin\theta \cdot |OB| = \sin\theta$

Note: $|BC| < |AB|$

$\Rightarrow \sin(\theta) = |BC| < |AB| = \theta$

$\Rightarrow \frac{\sin(\theta)}{\theta} < 1$

Note:

arclength of circle $<$ perimeter of polygon

$\therefore \theta = \text{arc } AC < |BE| + |AE|$

$< |ED| + |AE|$

$= |AD| = |OA| \tan(\theta)$

$= \tan\theta$

$$\therefore \quad \theta < \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \cos(\theta) < \frac{\sin\theta}{\theta}$$

$$\Rightarrow \cos(\theta) < \frac{\sin\theta}{\theta} < 1$$

As $\theta \to 0$ $\cos(\theta) \to \cos(0) = 1$

Thus by the squeeze thm

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

$$\frac{\cos\theta - 1}{\theta} = \frac{\cos\theta - 1}{\theta} \frac{\cos\theta + 1}{\cos\theta + 1}$$

$$= \frac{\cos^2\theta - 1}{\theta(\cos\theta + 1)} = \frac{-\sin^2\theta}{\theta(\cos\theta + 1)}$$

$$= \frac{-\sin\theta}{\theta} \cdot \frac{\sin\theta}{(\cos(\theta) + 1)}$$

$$\theta \to -1 \cdot 0 = 0$$

$$\therefore \quad f'(x) = \cos(x)$$

eg. $f(x) = x^3 \sin(x)$ $f' =$

$$\frac{d}{dx}\cos(x) = \sin(x)$$

$$\therefore \frac{d}{dx}\tan(x) = \frac{d}{dx}\left(\frac{\sin}{\cos}\right)$$

$$\csc \xrightarrow{D} \csc\cot$$
$$\sec \rightarrow \sec\tan$$
$$\cot \rightarrow \csc^2$$

eg. $f(x) = \dfrac{\sec}{1+\tan}$ $f' = ?$

eg. $\lim_{x\to 0} \dfrac{\sin 2x}{4x}$

eg. $\lim x\cot(x)$