§3.9 Related Rates

eg: Air is being pumped into a balloon so that its volume increases at a rate of $100\ cm^3/s$. How fast is the radius of the balloon increasing when the diameter is 50cm?

Know: rate of change of vol. is $100\ cm^3/s$.

Want: Rate of change of radius when the radius is 25cm.

Notation: V = volume, r = radius

$\frac{dV}{dt}$ = rate of change of vol. $\quad \frac{dr}{dt}$ = rate of change of radius,

Know: $\frac{dV}{dt} = 100\ cm^3/s$ **Want:** $\left.\frac{dr}{dt}\right|_{r=25}$

Relation: Chain Rule

$$V = \frac{4}{3}\pi r^3 \ cm^3/s$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \ cm/s$$

Solve:

$$\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt} = \frac{1}{4\pi r^2}(100)\ cm/s$$

$$= \frac{25}{\pi r^2}\ cm/s$$

Thus, $\left.\frac{dr}{dt}\right|_{r=25} = \frac{25}{\pi(25)^2}\ cm/s = \frac{1}{25\pi}\ cm/s$

Strategy:

1.) Read carefully!

2.) Draw a diagram or picture.

3.) Assign symbols to all quantities which are functions of time.

4.) Express "known"s and "want"s in terms of derivatives of above quantities

5.) Find a relation between quantities whose derivatives you would like to relate. (Possibly use geometry to eliminate one of the variables).

6.) Use the chain rule to diff the relation w.r.t. t.

7.) Substitute & solve for the desired derivative.

8.) Write a conclusion sentence!

eg. A ladder 10 ft. long rests against a vertical wall, If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

eg. A water tank has the shape of an inverted circular cone w/ base radius 2m and height 4m. If water is being pumped into the tank at a rate of $2 m^3/s$, find the rate at which the water level is rising when the water is 3ft. deep.

<u>Eg.:</u> Car A is travelling west at 50 m.p.h. and car B is travelling north at 60 m.p.h. Both are heading for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?

e.g. A man walks along a straight path at 4 ft/s. A search light is on the ground 20 ft from the path and is kept focused on the man. At what rate is the light rotating when the man is 15 ft. from the pt. on the path closest to the light?