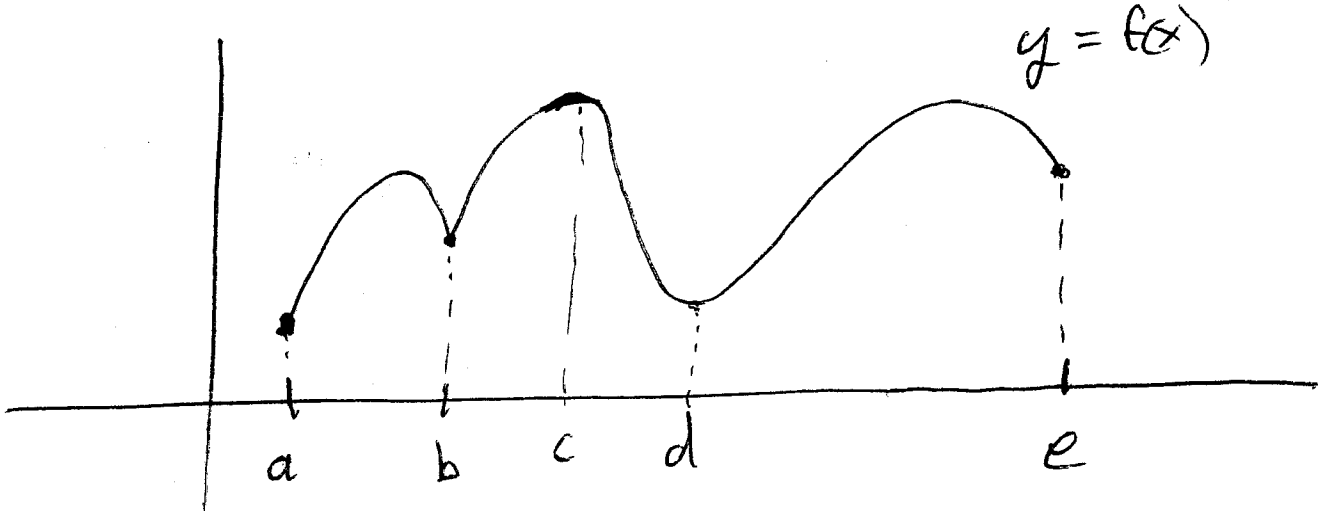
# § 4.1 Max. & Min. Values.

- **Def:** a fc, f has an *absolute maximum* (or global maximum) at c provided $f(c) \geq f(x)$ $\forall x \in$ Domain(f). The number f(c) is called the *maximum value of f* on its domain. Similarly f has an *absolute minimum* at c (or *global minimum*) provided $f(c) \leq f(x)$ $\forall x \in D$ ($D$ = domain(f)). The number f(c) is called the minimum value of f on D. The max and min, values of f are called the extreme values of f.




f has domain $D = [a, e]$.

f has a ~~---~~ global max at ____.

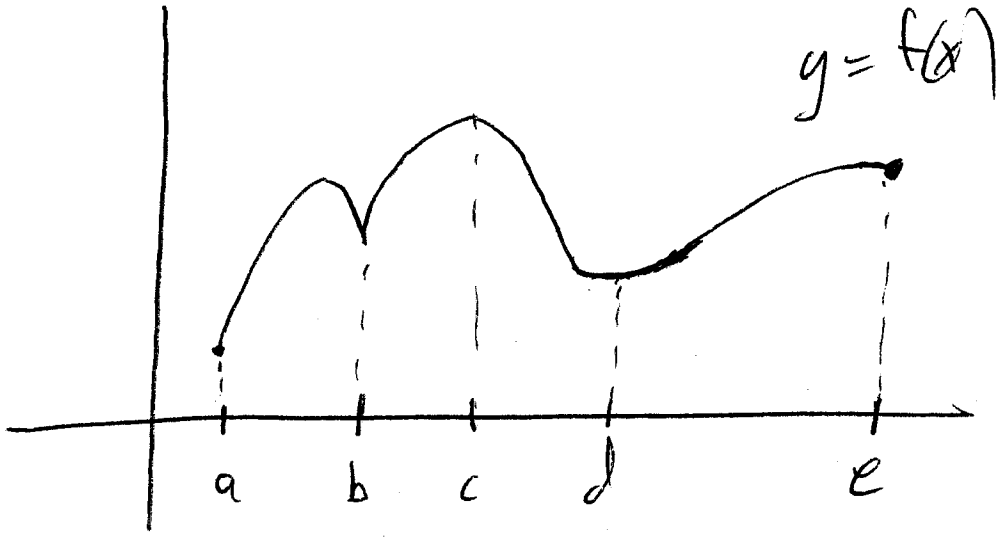
f has a global min at ____.

The max value of f is ____.

The min value of f is ____.

- **Def:** 1) $f$ has a local minimum (or relative minimum) at $c$ provided $f(c) \leq f(x)$ for $x$ near $c$.
(i.e. $\exists$ an open interval $I$ around $c$ and $f(c) \leq f(x)$ $\forall$ $x \in I$)

2) similarly $f$ has a local maximum (or relative maximum) at $c$ provided $f(c) \geq f(x)$ for all $x$ near $c$.



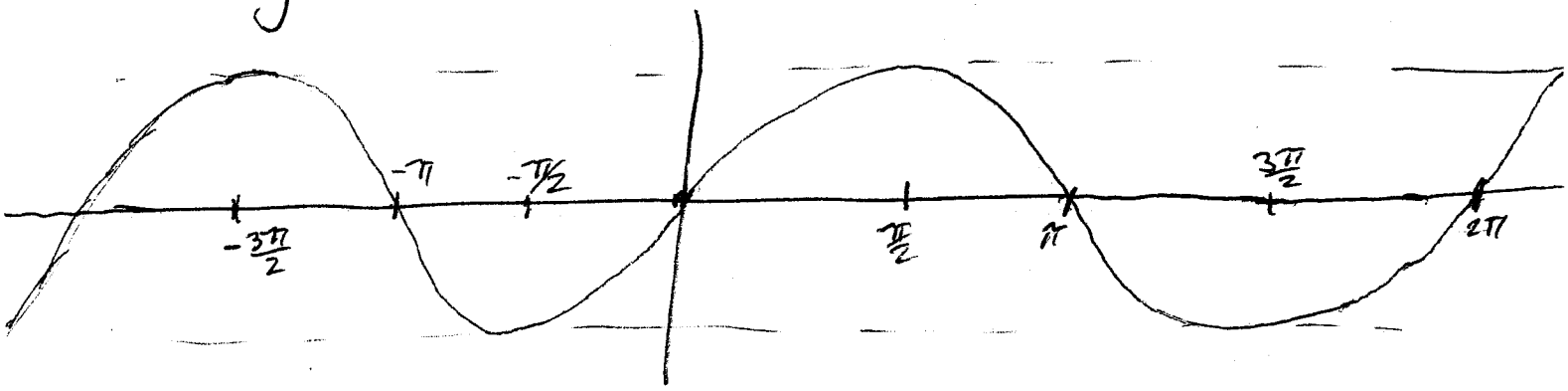

$f$ has a global min at _____.

$f$ has local min.'s at _____.

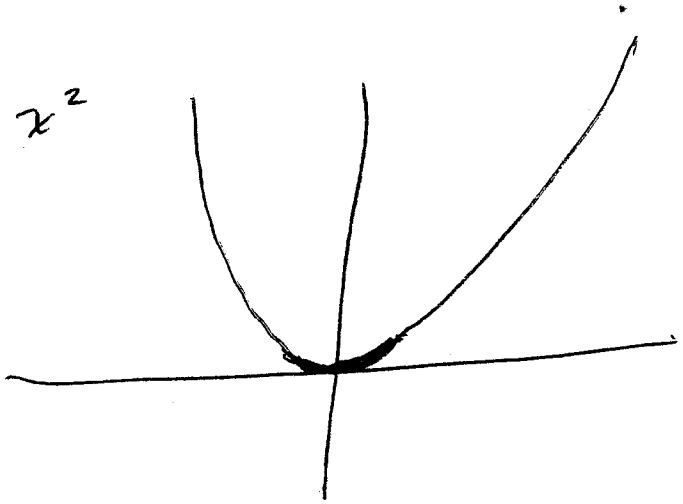
$f$ has a global max at _____.

$f$ has local max.'s at _____.

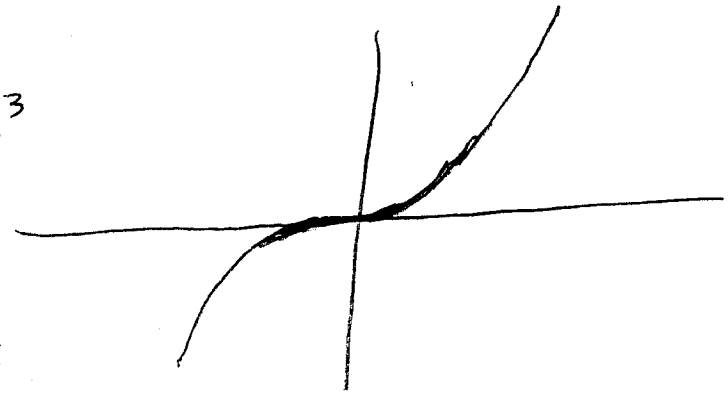
eg.: $y = \sin(x)$




eg.: $y = x^2$

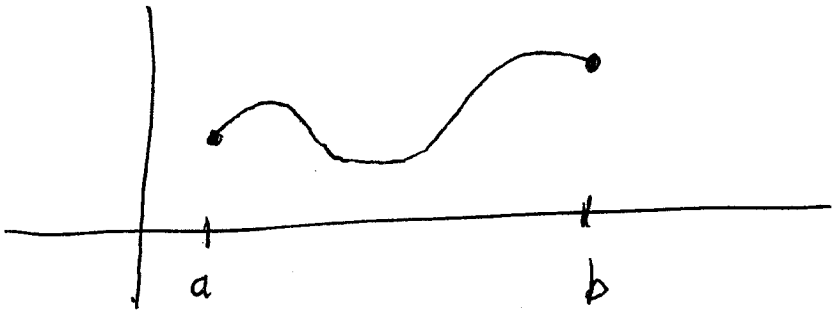


eg.: $y = x^3$

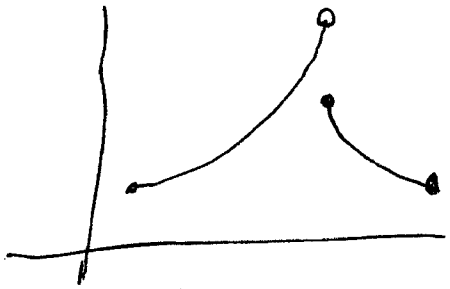


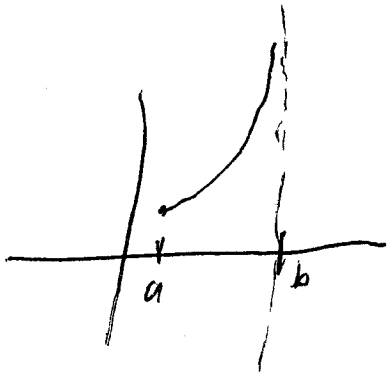
## Extreme Value Thm:

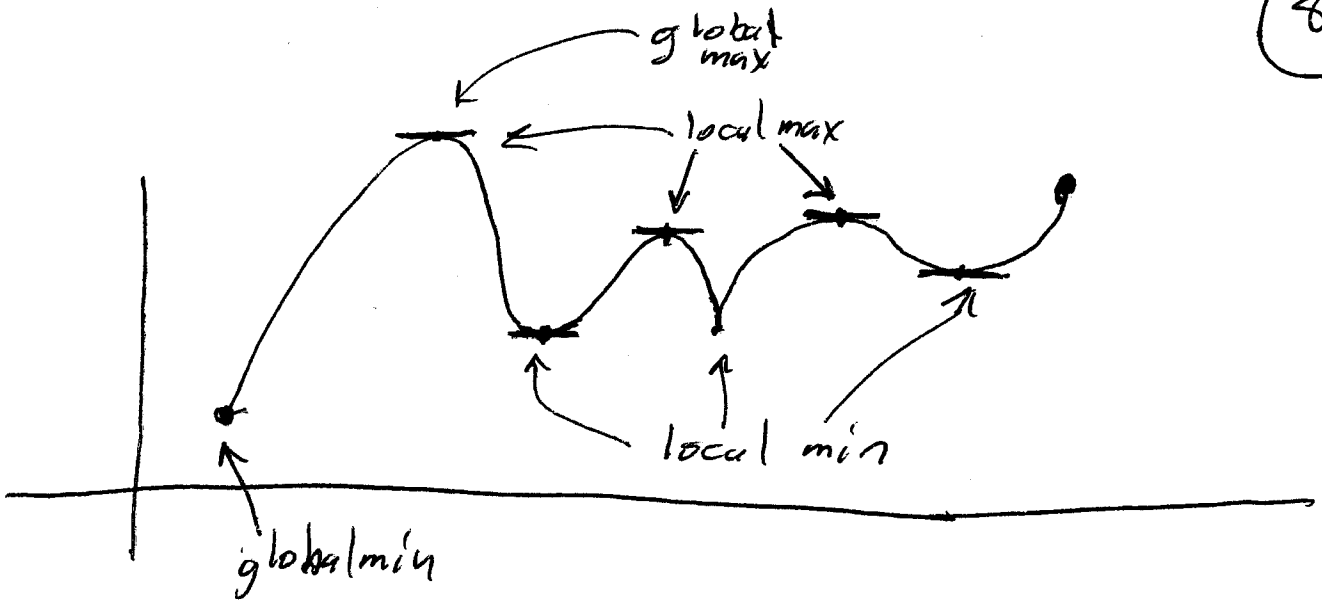
If $f$ is cont. on a closed interval $[a, b]$ then $f$ attains an abs. max. value $f(c)$ and an abs. min. value $f(d)$ for some $a \le c, d \le b$.




- Failure of E.V.T. when some conditions are not met







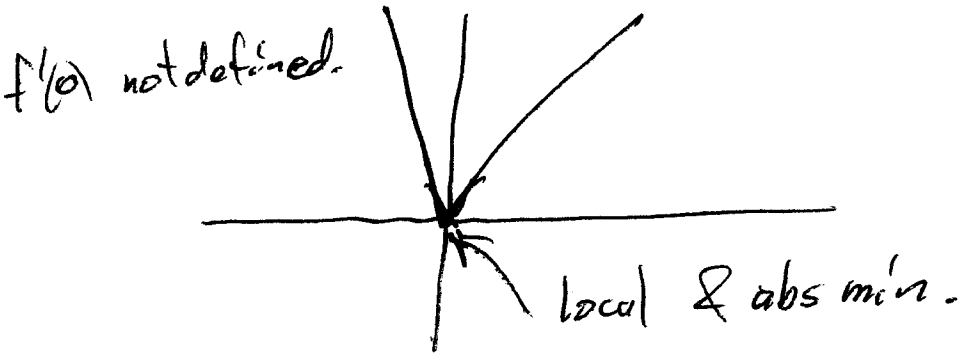

- **Fermat's Thm.:**

If $f$ has a local max or min at $c$ and if $f'(c)$ exists then $f'(c) = 0$.

**eg.:** $f(x) = x^2$

**eg.:** $f(x) = x^3$

eg. $f(x) = |x|$




- **Def:** A critical number of a function $f$ is a number $c$ in the domain of $f$ s.t. either $f'(c)=0$ or $f'(c)$ does not exist.

eg. Find the critical numbers of

$$f(x) = x^{3/5}(4-x)$$

• **Fact:** If $f$ has a local extreme at $c$ then $c$ is a critical number.

• **Closed Interval Method:**

To find the abs. extremes of a cont. fc. $f$ on a closed interval $[a, b]$

1) Find the values of $f$ at the critical numbers $c \in (a, b)$
2) Find $f(a)$ & $f(b)$.
3) The largest val. is the obs. max
   The smallest val. is the obs. min.

**eg.** Find the extreme values of

$$f(x) = x^3 - 3x^2 + 1$$

on $\left[-\frac{1}{2}, 4\right]$.