§4.10 Antiderivatives.

- **Def.** a function $F(x)$ is called an antiderivative of $f(x)$ on an interval I provided that

$$F'(x) = f(x) \quad \forall x \in I.$$

- **eg.** $F(x) = \frac{1}{2}x^2$ is an ad. of $f(x) = x$ on $(-\infty, \infty)$

$G(x) = \frac{1}{2}x^2 + 10$ is another.

In fact, $H(x) = \frac{1}{2}x^2 + C$ is an ad. for $f(x) = x$ on $(-\infty, \infty)$ for any constant C.

- **Fact:** If F & G are both ad's of $f(x)$ on I, then $F - G = C$ where C is a constant.

- **Def:** If F is an ad. of f on I, then the most general ad. for $f(x)$ is

$$G(x) = F(x) + C,$$

where C is an arbitrary constant.

eg: Find the most general antiderivative

a) $f(x) = \cos(x)$

b) $f(x) = x^4$

c) $f(x) = x^n \quad (n \geq 0)$

d) $f(x) = x^{-5}$

e) $f(x) = \frac{1}{2\sqrt{x}}$

If $F' = f$ & $G' = g$

Function	Anti
$c\,f(x)$	$c\,F(x)$
$f(x)+g(x)$	$F(x)+G(x)$
$x^n\ (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$\cos(x)$	$\sin(x)$
$\sin(x)$	$-\cos(x)$
$\sec^2(x)$	$\tan(x)$
$\sec(x)\tan(x)$	$\sec(x)$

e.g.: Find all functions $G(x)$ s.t.

$$G'(x) = 5\cos(x) + \frac{2x^3 + 3\sqrt{x}}{x^2}$$

Def: An equation involving derivatives of functions is a *differential equation*.

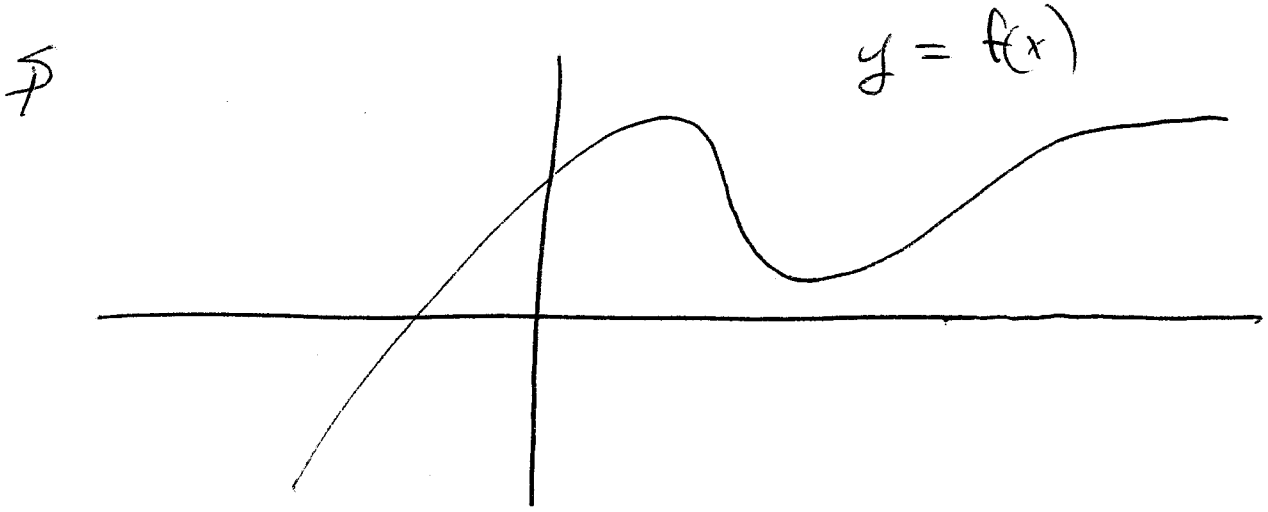
eg. Find $f(x)$ if

$f'(x) = x\sqrt{x} + 2$ and $f(1) = 3$.

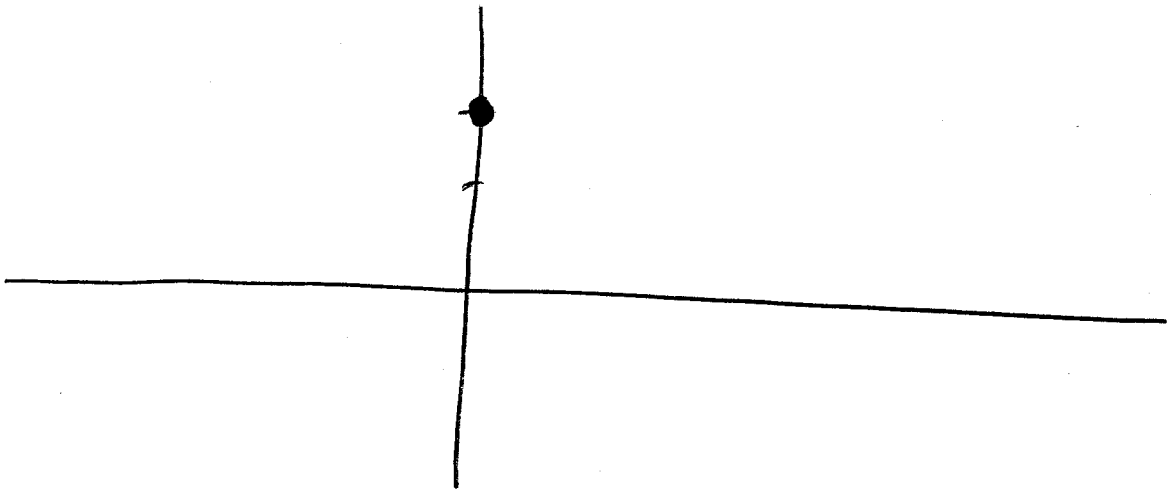
eg.: Find $f(x)$ if

$f''(x) = 12x^2 + 6x - 4$ and $f(0) = 4$ & $f(1) = 1$

Geometry of Antiderivatives.



Sketch the graph of an a.d. F of f. given that F(c) = 2



eg: $f(x) = \sqrt{1+x^3} - x$

Sketch the graph of the ad. ant. F of f w/ $F(-1) = 0$.