§4.3

Inc / Dec Test:

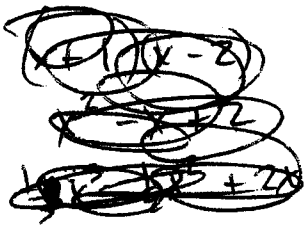
a) $f'(x) > 0$ on $I \Rightarrow f$ is inc. on I

b) $f'(x) < 0$ on $I \Rightarrow f$ is dec. on I

Ex: Where is

$$f(x) = x^3 - 3x^2 - 12x + 10$$

inc / dec. ?



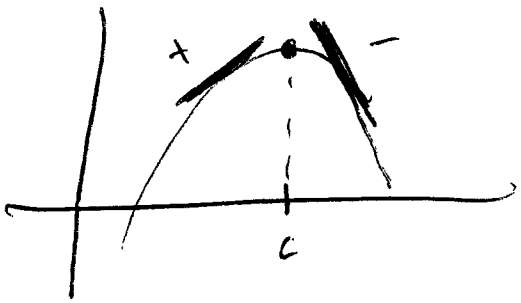
1st derivative test.

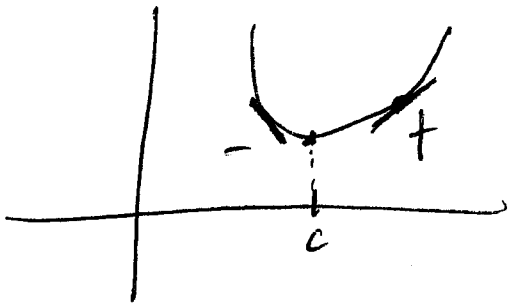
- If c is a critical # for f and f is cont.

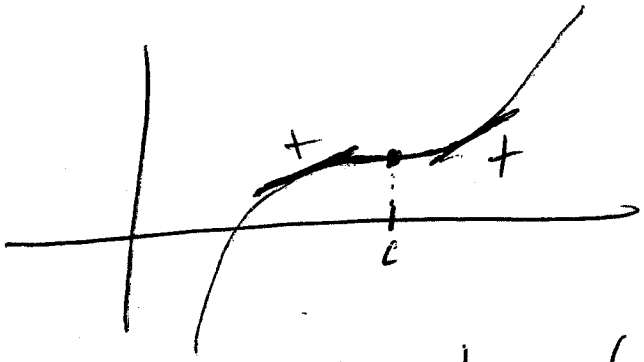
a) If f' changes from + to − at c then $f(c)$ is a local max

b) If f' changes from − to + at c then $f(c)$ is a local min.

c) No sign change $\Rightarrow$ no local extreme







eg: Find the local extrema of

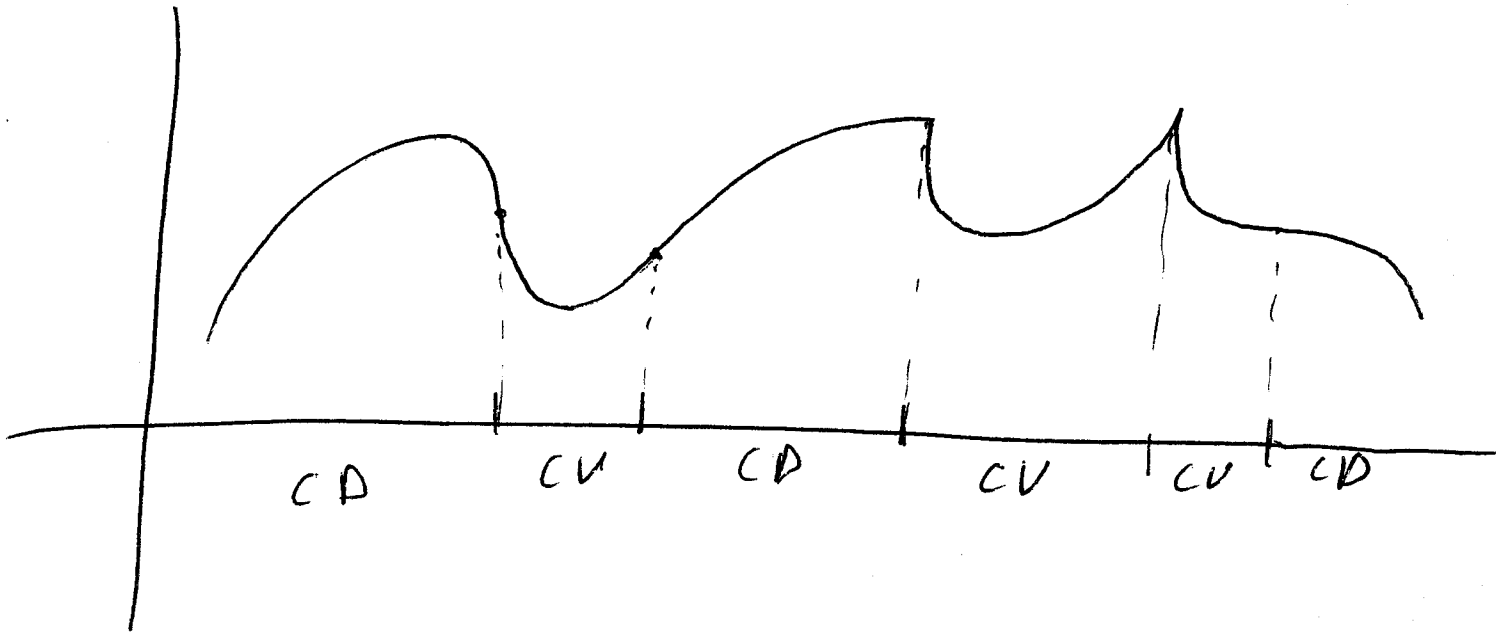
$$f(x) = 2x^3 - 3x^2 - 12x + 10.$$

eg: Find the local max/min values of

$$f(x) = \cos(x) + x \quad \text{on } [0, 2\pi]$$

~~Def~~: If the graph of f lies above its tangents ~~this is~~ on I then it is concave up on I.

If " " " " " below ' ' ' ' I then it is concave down on I.



Concavity Test:

a) $f''(x) > 0 \ \forall x \in I \Rightarrow f$ is C.U. on I

b) $f''(x) < 0 \ \forall x \in I \Rightarrow f$ is C.D. on I.

eg. Where is
$f(x) = 2x^3 + 9x + 12x$
cu & CD.

Def: Inf. Pt.: t cont & changes concavity

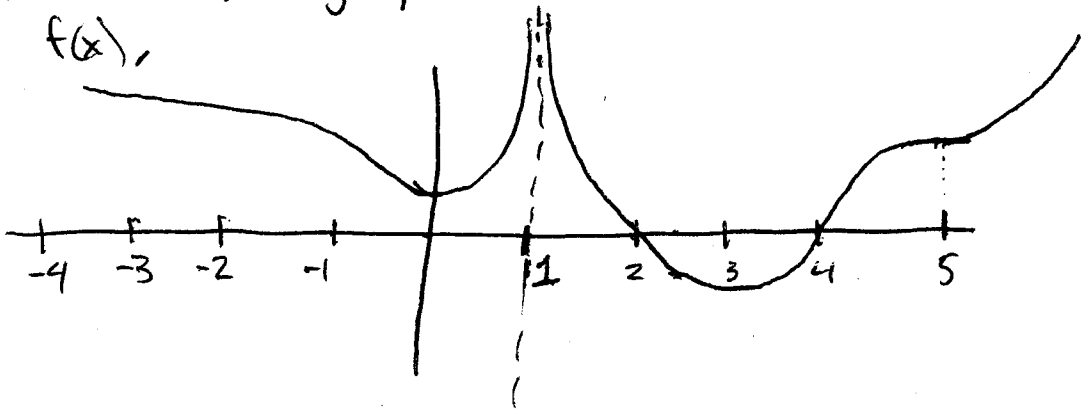
2nd Derivative test:
f'' cont near c
1) $f'(c)=0$ & $f''(c)>0 \Rightarrow$ a min
2) $f'(c)=0$ & $f''(c)<0 \Rightarrow$ max.

eg. Let $f(x) = x^4 - 4x^3$. Find pt's of inflection, concavity, local max/min. sketch the graph

eg: $f(x)=x^{2/3}(6-x)^{1/3}$. sketch the graph.

Quiz #11:

1.) Below is the graph of the first derivative of $f(x)$.



(2pt.) a.) what are the intervals of inc./dec.?

	$(-\infty,1)$	$(1,2)$	$(2,4)$	$(4,\infty)$
f'	+	+	−	+
f	inc.	inc.	dec	inc.

b.) Suppose that $f(1)=2$. What are the critical numbers?

(1pt.) $x=1,2,4$

(2pt.) c.) Locate the local extrema of $f(x)$.

f has a local max at $x=2$.

f has a local min at $x=4$.

(2pt.) d.) On which intervals is $f(x)$ concave up / concave down?

	$(-\infty,0)$	$(0,1)$	$(1,3)$	$(3,5)$	$(5,\infty)$
f''	−	+	−	+	+
f	C.D.	C.U.	C.D.	C.U.	C.U.

CD = concave down

CU = concave up

(2pt.) e.) Identify the inflection points.

f has inflection points at $x=0,1,3$.

(1pt.) f.) Sketch the graph of $f(x)$, given that $f(1)=0$.

ONE POSSIBLE Graph