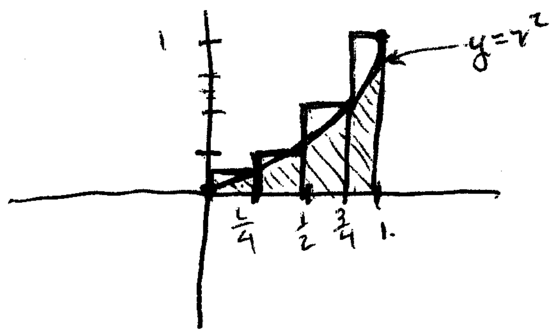
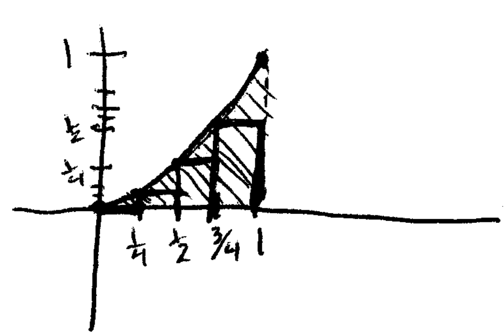
- eg. A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 ft above the ground. Find its height above the ground t seconds later. When does it reach its maximum height? When does it hit the ground?

§ 5.1 Areas & Distances.

eg.: Approximate the are under the curve $y = x^2$ from $x=0$ to $x=1$.





$R_4 = \sum_{i=1}^{4}$ area of i^{th} rectangle

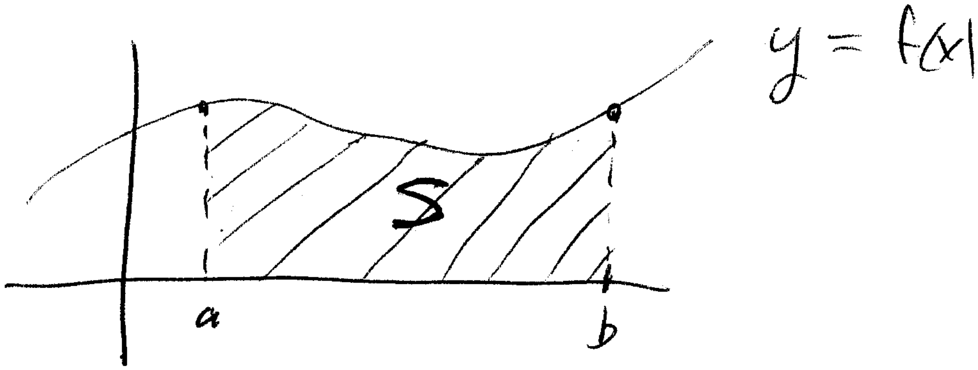
$L_4 = \sum_{i=1}^{4}$ area of i^{th} rect.

eg.: Compute the area in the first example.

<u>Useful formula:</u> $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

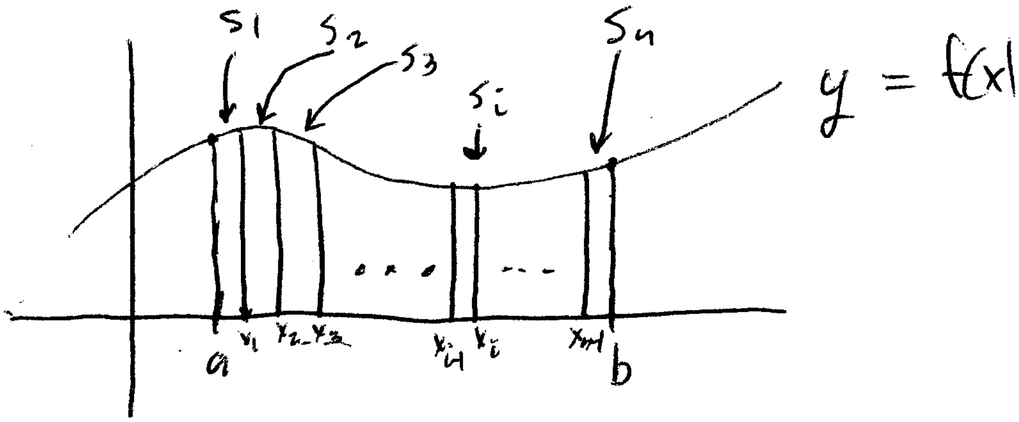
$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

- More generally,



What is the area of S?

Step 1: Divide S into n strips



Area of S $= \sum_{i=1}^{n}$ area of S_n.

Width of a strip:

$$\Delta x = \frac{b-a}{n}$$

- Note: We have subdivided $[a b]$ into n subintervals

$$[x_0, x_1]\quad [x_1, x_2], \dots, [x_{n-1}, x_n]$$

a $\rightarrow x_0$ $\qquad$ $x_n \leftarrow$ b

$x_0 = a$

$x_1 = a + \Delta x$

$x_2 = x_1 + \Delta x = a + 2\Delta x$

$x_3 = x_2 + \Delta x = a + 3\Delta x$

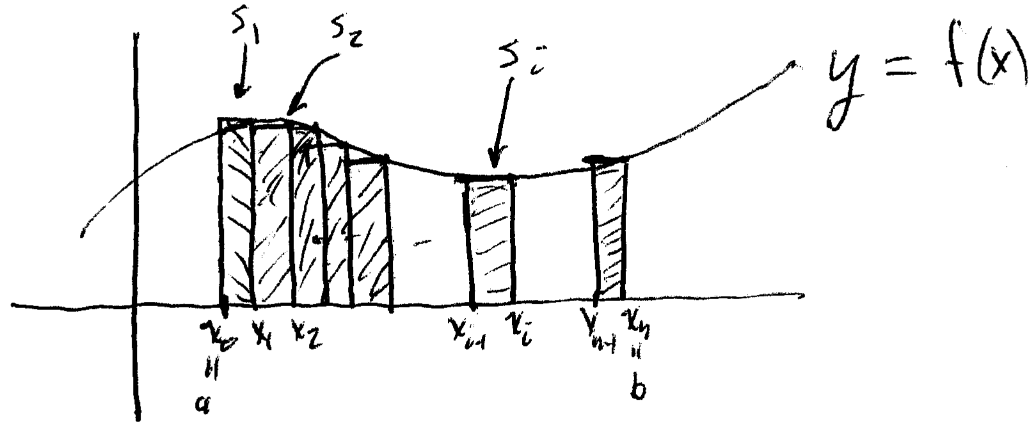
$\vdots$

$x_i = a + i\,\Delta x$

$\vdots$

$x_n = a + n\,\Delta x = a + n\left(\frac{b-a}{n}\right) = b$

<u>Step 2</u>: Approximate the area of S_i by the area of the rectangle of width Δx and height $f(x_i)$



$\text{Area}(S_i) \approx \Delta x\, f(x_i)$

$\text{Area}(S) \approx \underline{\sum_{i=1}^{n} \Delta x\, f(x_i)} \rightarrow R_n$

Def: The area of the region S that lies under the graph of the cont. fc. f(x) is given by

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\left[\sum_{i=1}^{n} f(x_i)\,\Delta x\right]$$

Fact: We would get the same value of A if we had used left end points, or in fact if we had taken the height of our rectangle to be $f(x_i^*)$ where x_i^* is any point in $[x_{i-1}, x_i]$.

So,

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x \qquad \text{(right endpoints)}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_{i-1})\,\Delta x \qquad \text{(left endpoints)}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

e.g.: Let A be the area of the region that lies under $y = \cos(x)$ between $x = 0$ and $x = b$ where $0 \le b \le \frac{\pi}{2}$.

a) Using right end points find an expression for A as a limit.

b) Estimate the area when $b = 2$ by using 4 rectangles with height given by $\cos(x_i^*)$ where x_i^* is the mid point.

The distance problem.

§ the velocity of an object in ft/sec is given at the following times

time	0	5	10	15	20	25	30
V	17	21	24	29	32	31	28

Approximate the distance travelled

Recall

dist. = vel. × time

When $0 \le t \le 5$ $\quad 17 \le V \le 21$

$\Rightarrow$ dist $\approx 5 \times 17 = 85$

when $5 \le t \le 10$ $\quad 21 \le V \le 24$

dist $\approx 21 \times 5 = 105$

Continuing we get that the total distance travelled from $t=0$ to $t=30$ is approximately

$$\text{dist} \approx \underbrace{5\times 17}_{0\le t\le 5} + \underbrace{5\times 21}_{5\le t\le 10} + \underbrace{5\times 24}_{10\le t\le 15} + \underbrace{5\times 29}_{15\le t\le 20} + \underbrace{5\times 32}_{20\le t\le 25} + \underbrace{5\times 31}_{25\le t\le 30}$$

$$= 5\,(17+21+24+29+32+31)$$

$$= 5(154) = 770 \text{ ft.}$$

In general if an object has velocity $v(t)$. Then the distance travelled from $t = a$ to $t = b$ is given by

$$d = \lim_{n\to\infty} \sum_{i=1}^{n} v(t_{i-1})\,\Delta t$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} v(t_i)\,\Delta t$$

This is simply the area under $y = v(t)$ from a to b.