Indefinite Integrals.

- **Notation:** We typically denote the antiderivative of $f(x)$ by $\int f(x)\,dx$ and call this the indefinite integral.
  (i.e. $F(x) = \int f(x)\,dx$ means that $F'(x) = f(x)$)

eg.

$$\int x^2\,dx = \frac{x^3}{3} + C.$$

Note:

1.) Definite integrals are numbers.

2.) Indefinite integrals are functions.

3.) $\displaystyle\int_a^b f(x)\,dx = \left[\int f(x)\,dx\right]_a^b$

Table of Integrals:

$$\int c\,f(x)\,dx = c\int f(x)\,dx \qquad \int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C \qquad \int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \sin(x)\,dx = -\cos(x) + C \qquad \int \cos(x)\,dx = \sin(x) + C$$

$$\int \sec^2(x)\,dx = \tan(x) + C \qquad \int \csc^2(x)\,dx = -\cot(x) + C$$

$$\int \sec(x)\tan(x)\,dx = \sec(x) + C \qquad \int \csc(x)\cot(x)\,dx = -\csc(x) + C$$

Note: We adopt the convention that the indefinite integral is defined <u>only</u> on an interval on which the integrand is continuous.

<u>eg.</u> we write
$$\int \frac{1}{x^2}\,dx = -\frac{1}{x} + C$$
with the understanding that this function is only defined on $(0,\infty)$ or on $(-\infty,0)$.

<u>Recall</u>: The most general a.d. of $\frac{1}{x^2}$ is
$$F(x) = \begin{cases} \frac{-1}{x} + C_1 & x < 0 \\ \frac{-1}{x} + C_2 & x > 0 \end{cases}$$

<u>eg.</u> what is $\int (10x^4 - 2\sec^2(x))\,dx$

eg.: $\int \frac{\cos(\theta)}{\sin^2(\theta)}\, d\theta =$

eg. $\int_0^{12} \left(x - 12\sin(x)\right) dx =$

eg. $\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} \, dt =$

Note. If $F(x)$ is a diff. fc. on $[a, b]$ then we can write

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

1) $F'(x)$ denotes the instantaneous rate of change in F w.r.t. x

2) $F(b) - F(a)$ is the net change in F as x changes from a to b.

- Net Change Theorem:

The integral of the instantaneous rate of change of F w.r.t. x is equal to the net change in F as x changes from a to b.

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

Examples:

(1) If $V(t)$ is the volume of water in a reservoir at time t, then $V'(t)$ is the rate at which the volume is changing at time t. So,

$$\int_{t_1}^{t_2} V'(t)\,dt = V(t_2) - V(t_1)$$

is the net change in volume from time t_1 until time t_2

(2) If an object moves along a straight line w/ position fc, $s(t)$, then its velocity function is $v(t) = s'(t)$. Thus,

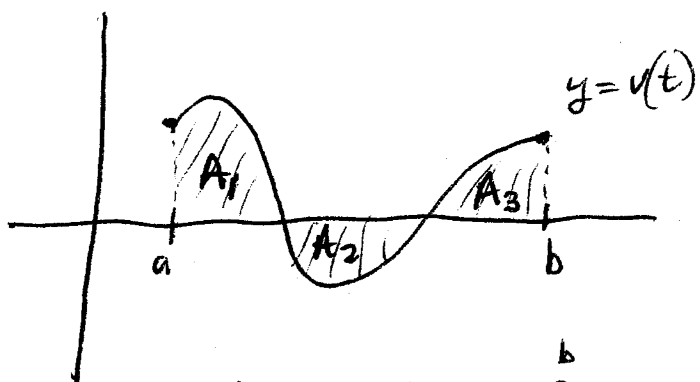
$$\int_{t_1}^{t_2} v(t)\,dt = s(t_2) - s(t_1)$$

is the net change in position of the object frome time t_1 to time t_2.
This is also called displacement.

- Note: If we want the distance travelled by the object,

$$\int_{t_1}^{t_2} |v(t)|\, dt = \text{total dist. travelled.}$$

- eg:



$$\text{dist travelled} = \int_a^b |v(t)|\,dt = A_1 + A_2 + A_3$$

$$\text{displacement} = \int_a^b v(t)\,dt = A_1 - A_2 + A_3$$

- Note: These are not the same.

Note:

$$a(t) = v'(t)$$

$$\Rightarrow \int_{t_1}^{t_2} a(t)\,dt = \text{net change in velocity.}$$

eg.: $v(t) = t^2 - t - 6$ m/s

a) Find the displacement during the time period $1 \le t \le 4$.

b) Find the distance travelled during this time