§5.5 The Substitution Rule.

eg: Evaluate $\int 2x\sqrt{1+x^2}\,dx$.

Note: We don't have a formula for this.

Solution:

Let $u = 1+x^2$

Then $du = 2x\,dx$

Thus,

$$\int \sqrt{1+x^2}\,(2x\,dx) = \int \sqrt{u}\,du$$

$$= \tfrac{2}{3}u^{3/2} + C$$

$$= \tfrac{2}{3}(1+x^2)^{3/2} + C$$

Check:

$$\frac{d}{dx}\left(\tfrac{2}{3}(1+x^2)^{3/2} + C\right) = (1+x^2)^{\frac{1}{2}}(2x)$$

$$= 2x\sqrt{1+x^2} \quad \checkmark$$

Substitution Rule.

If $u = g(x)$ is a diff. fc. whose range is in an interval I and if f is cont. on I, then

$$\int f(g(x))\, g'(x)\, dx = \int f(u)\, du$$

To see this:

Suppose that $F(x)$ is an ad. for $f(x)$, i.e. $F'(x) = f(x)$. Then by the chain rule we have

$$\frac{d}{dx}\left(F(g(x))\right) = F'(g(x))\, g'(x)$$
$$= f(g(x))\, g'(x)$$

Thus $F(g(x))$ or $F(u)$ is an antiderivative for

$$f(g(x))\, g'(x).$$

eg: Evaluate $\int x^2 \cos(x^3+2)\,dx$

eg. $\int \sqrt{5x+2}\,dx$

eg. $\int \frac{x^2}{\sqrt{1+4x^3}}\,dx$

eg: $\int \cos(10x)\,dx$

eg: $\int \frac{x^5}{\sqrt{1+x^2}}\,dx$

Substitution Rule for Definite Integrals.

If g' is cont on $[a,b]$ and f is cont. on the range of $u=g(x)$ then

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

e.g.: $\displaystyle\int_0^4 \sqrt{2x+1}\, dx$

e.g.: $\displaystyle\int_1^2 \frac{dx}{(3-5x)^2}$

<u>Symmetry:</u> ~~S~~ f is cont. on $[-a, a]$

a) If f is even then,

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

b) If f is odd then

$$\int_{-a}^{a} f(x)\,dx = 0$$

<u>eg.:</u> $\int_{-2}^{2} (x^6+1)\,dx$

<u>eg.:</u> $\int_{-1}^{1} \frac{\tan(x)}{(1+x^2+x^4)}\,dx$