

§5.5 The Substitution Rule.

eg.: Evaluate $\int 2x \sqrt{1+x^2} dx$,

Note: We don't have a formula for this.

Solution:

$$\text{Let } u = 1+x^2$$

$$\text{Then } du = 2x dx$$

Thus,

$$\int \sqrt{1+x^2} (2x dx) = \int \sqrt{u} du$$

$$= \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{3} (1+x^2)^{3/2} + C$$

Check:

$$\frac{d}{dx} \left(\frac{2}{3} (1+x^2)^{3/2} + C \right) = (1+x^2)^{1/2} (2x)$$

$$= 2x \sqrt{1+x^2} \quad \checkmark$$

• Substitution Rule.

If $u = g(x)$ is a diff. fc, whose range is in an interval I and if f 's cont, on I , then

$$\int f(g(x)) g'(x) dx = \int f(u) du$$

• To see this:

Suppose that $F(x)$ is an ant. for $f(x)$, i.e. $F'(x) = f(x)$.
Then by the chain rule we have

$$\begin{aligned} \frac{d}{dx} (F(g(x))) &= F'(g(x)) g'(x) \\ &= f(g(x)) g'(x) \end{aligned}$$

~~Thus~~ Thus $F(g(x))$ or $F(u)$ is an antiderivative for $f(g(x)) g'(x)$.

eg: Evaluate $\int x^2 \cos(x^3 + z) dx$

eg. $\int \sqrt{5x+z} dx$

-eg. $\int \frac{x^2}{\sqrt{1+4x^3}} dx$

ex: $\int \cos(10x) dx$

ex: $\int \frac{x^5}{\sqrt{1+x^2}} dx$

Substitution Rule for Definite Integrals.

If g' is cont on $[a, b]$ and f is cont. on the range of $u = g(x)$ then

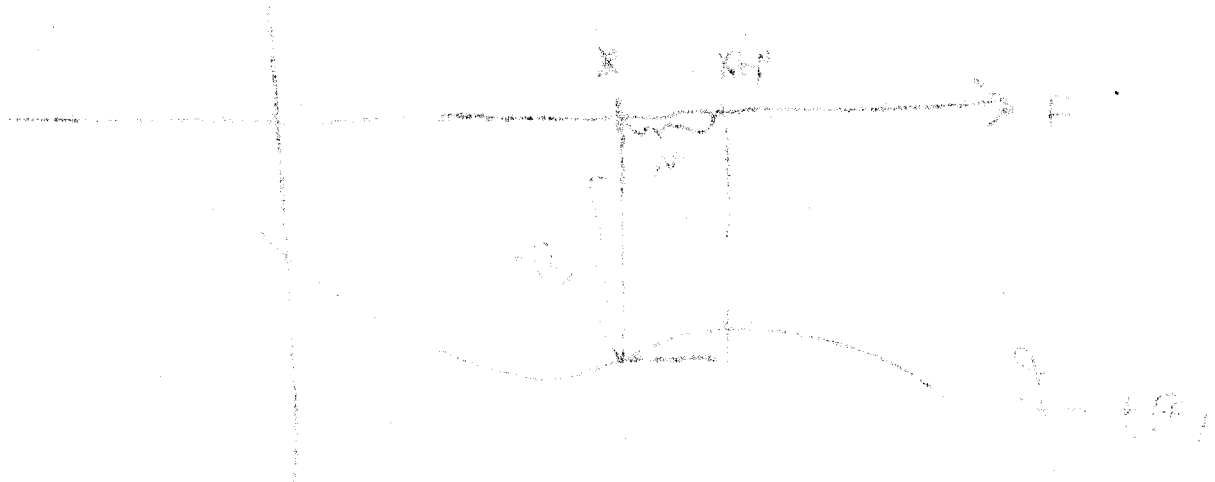
$$\int_a^b f(g(x)) g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

eg.: $\int_0^4 \sqrt{2x+1} \, dx$

eg.: $\int_1^2 \frac{dx}{(3-5x)^2}$

NOTE:

$(3-5x) \rightarrow 0 \Rightarrow x = \frac{3}{5}$
 we change
 limits of integration



Symmetry: If f is cont. on $[-a, a]$

a) If f is even then,

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

b) If f is odd then

$$\int_{-a}^a f(x) dx = 0$$

eg.: $\int_{-2}^2 (x^6 + 1) dx$

eg.: $\int_{-1}^1 \frac{\tan(x)}{(1+x^2+x^4)} dx$