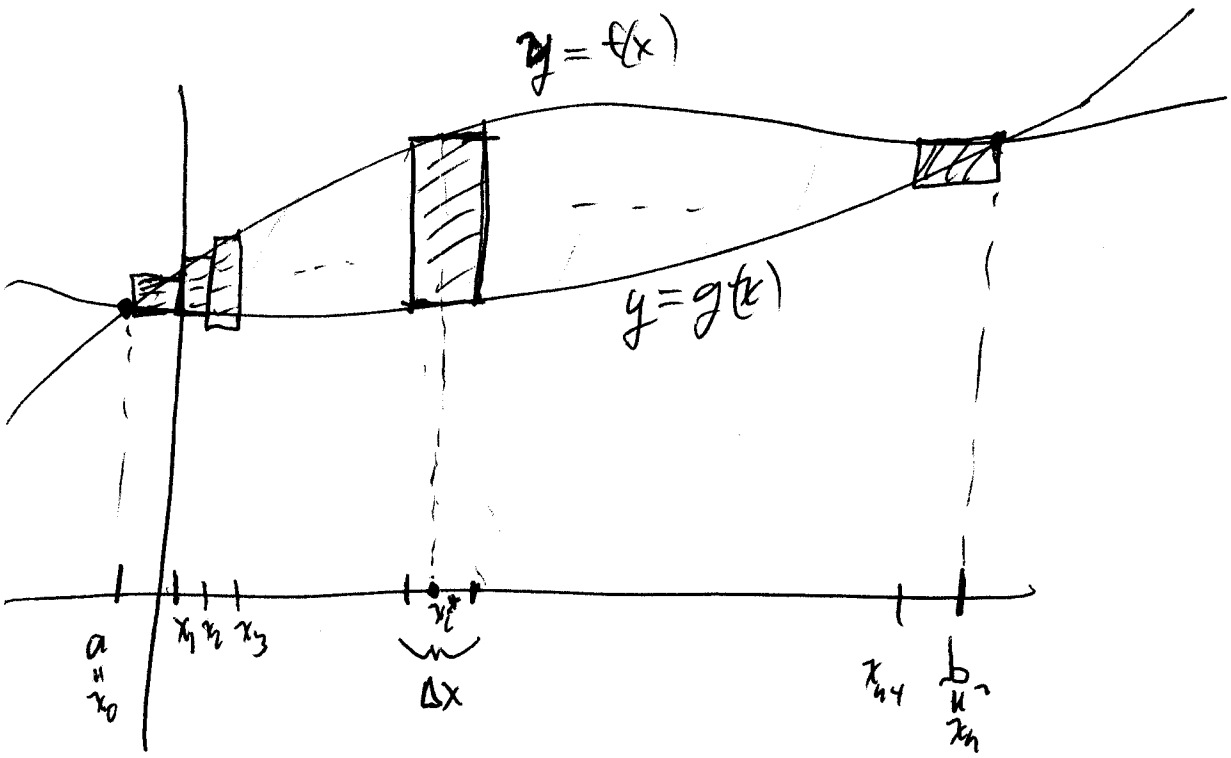
§ 6.1 Areas between curves.



Area of the i^{th} rectangle $= \Delta x \left(f(x_i^*) - g(x_i^*)\right)$

$$\text{Area} = \sum_{i=1}^{n} \text{Area of } i^{th} \text{ rectangle}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(f(x_i^*) - g(x_i^*)\right) \Delta x$$

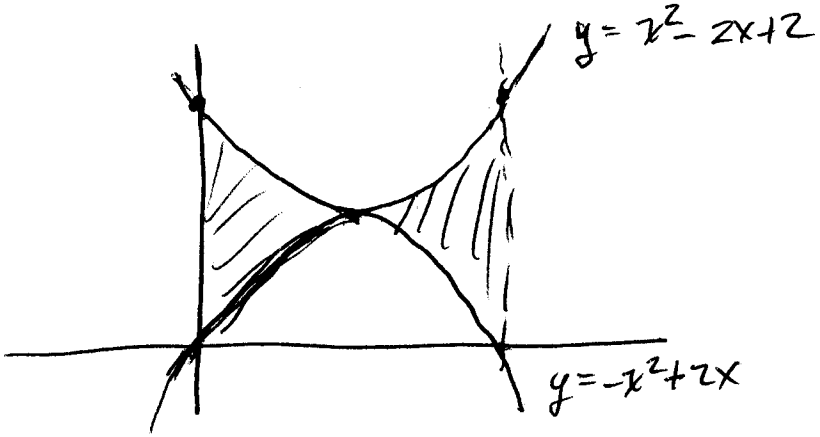
$$= \int_a^b \left(f(x) - g(x)\right) dx$$

Note: We are assuming that $f(x) \le g(x)$ for all x in (a, b).

In general, the area between two curves $y = f(x)$ and $y = g(x)$ and between $x = a$ and $x = b$ is given by

$$A = \int_a^b |f(x) - g(x)| \, dx$$

eg.



Compute the area bounded by

$y = x^2 - 2x + 2$

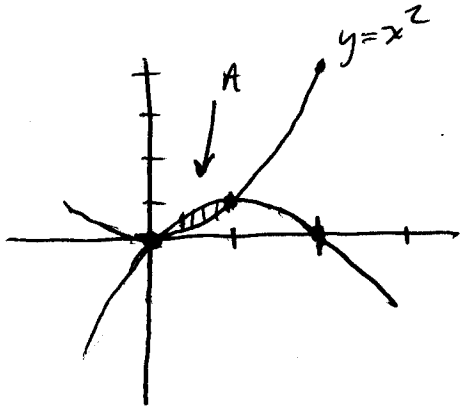
$y = -x^2 + 2x$

$x = 0$; $x = 2$

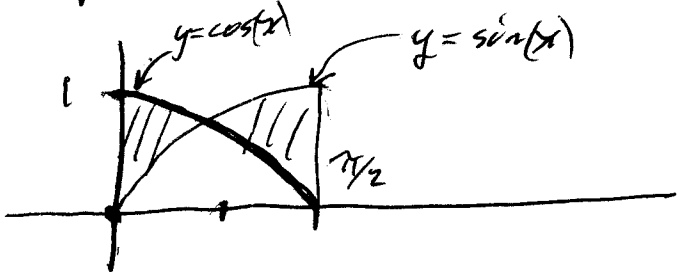
$$A = \int_0^2 [(x^2 - 2x + 2) - (-x^2 + 2x)] \, dx$$

$=$

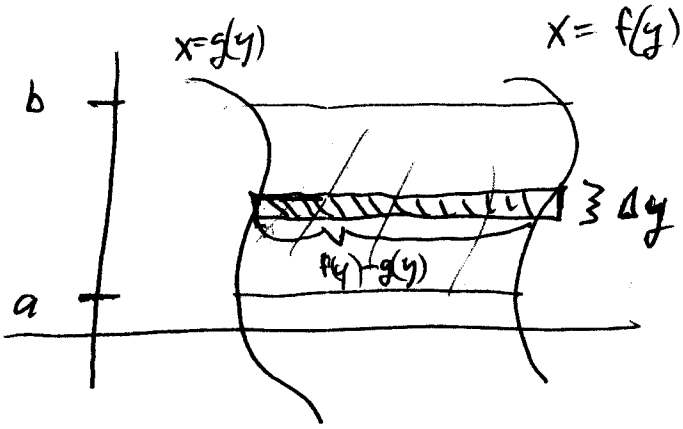
eg: Find the area enclosed by $y = x^2$ & $y = 2x - x^2$



eg. Find the area between $y = \sin(x)$ and $y = \cos(x)$ from $x = 0$ to $x = \frac{\pi}{2}$



Note: Sometimes it is more convenient to integrate w.r.t. y.



$$A = \lim_{n\to\infty} \sum_{i=1}^{n} (f(y)-g(y))\,\Delta y$$

$$= \int_a^b (f(y)-g(y))\,dy$$

Eg.: Compute the area enclosed by $y = x-1$ & $y^2 = 2x-6$