Key

Quiz #12: Let $f(x) = \dfrac{2x^2-2}{x^2-4}$.

a.) What is the domain of $f(x)$?
$(-\infty,-2) \cup (-2,2) \cup (2,\infty)$

b.) What are the x- and y-intercepts?
x-intercepts: $x = \pm 1$
y-int: $\frac{1}{2}$

c.) Is f even, odd or neither?
even

d.) What are the vertical asymptotes of $f(x)$? Justify your answer using one-sided limits.

$x=2$ b/c $\lim_{x\to 2^-} \frac{2(x+1)(x-1)}{(x+2)(x-2)} = -\infty$ $\lim_{x\to 2^+} \frac{2(x+1)(x-1)}{(x+2)(x-2)} = \infty$

$x=-2$ b/c $\lim_{x\to -2^-} \frac{2(x+1)(x-1)}{(x+2)(x-2)} = \infty$ $\lim_{x\to -2^+} \frac{2(x+1)(x-1)}{(x+2)(x-2)} = -\infty$

e.) Identify the critical points and intervals of increase/decrease for $f(x)$. Also indicate any local extremes.

$f'(x) = \dfrac{-12x}{(x^2-4)^2}$

Critical Pt: $x=0$
f' is undefined at $x=\pm 2$ but so is f.

	$(-\infty,-2)$	$(-2,0)$	$(0,2)$	$(2,\infty)$
f'	+	+	−	−
f	inc	inc	dec	dec

Local max val. of $f(0) = \frac{1}{2}$ at $x=0$.

f.) Identify intervals of concavity and inflection points.

$f''(x) = \dfrac{12(3x^2+4)}{(x^2-4)^3}$

f'' is undefined at ± 2 as is f & f', f'' is never 0.

	$(-\infty,-2)$	$(-2,2)$	$(2,\infty)$
f''	+	−	+
f	CU	CD	CU

No points of inflection b/c changes in concavity occur where f is not defined

g.) Identify horizontal asymptotes. Justify your answer with limits.

$\lim_{x\to\infty} \frac{2x^2-2}{x^2-4} = \lim_{x\to\infty} \frac{2-2/x^2}{1-4/x^2} = 2$

$\lim_{x\to-\infty} \frac{2x^2-2}{x^2-4} = \lim_{x\to-\infty} \frac{2-2/x^2}{1-4/x^2} = 2$

Thus, $y=2$ is the only horizontal asymptote.

h.) Sketch the graph of $y = f(x)$.