Quiz 14 Key

1.) Compute $\int_0^1 x\,dx$. You will need to use the identity $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

$$\Delta x = \frac{1-0}{n} = \frac{1}{n}$$

$$x_i = a + i\Delta x = i/n$$

$$\int_0^1 x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{n^2}$$

$$= \lim_{n\to\infty} \frac{1}{n^2} \sum_{i=1}^{n} i = \lim_{n\to\infty} \left(\frac{n^2+n}{2n^2}\right)$$

$$= \lim_{n\to\infty} \left[\frac{1+\frac{1}{n}}{2}\right] = \frac{1}{2}.$$

(2.) Use right endpoints and 4 rectangles to approximate $\int_0^4 \sqrt{x}\,dx$.

$n=4$

$$\Delta x = \frac{b-a}{4} = \frac{4-0}{4} = 1$$

$$x_i = a + i\Delta x = 0 + i\cdot 1 = i$$

$$\int_0^4 \sqrt{x}\,dx \approx \sum_{i=1}^{4} f(x_i)\Delta x = \sum_{i=1}^{4} f(i)\cdot 1$$

$$= \sum_{i=1}^{4} \sqrt{i} = \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4}$$