

2 pt's each.

MAT 106
Quiz #17
November 8, 2004

Name: Key

You may not use your notes. Please show all of your work. An answer without justification will receive little credit.

$$(1) \int \sec^2(\cos(x)) \sin(x) dx = - \int \sec^2(u) du = -\tan(u) + C = -\tan(\cos(x)) + C$$

$$\boxed{\begin{aligned} u &= \cos(x) \\ du &= -\sin(x) dx \end{aligned}}$$

$$(2) \int (x^2 + 2x + 3)^{-2/3} (x+1) dx = \frac{1}{2} \int u^{-2/3} du = \frac{3}{2} u^{1/3} + C = \frac{3}{2} (x^2 + 2x + 3)^{1/3} + C$$

$$\boxed{\begin{aligned} u &= x^2 + 2x + 3 \\ du &= 2x + 2 = 2(x+1) dx \end{aligned}}$$

$$(3) \int \frac{x^3 dx}{\sqrt{x^2+1}} =$$

$$\boxed{\begin{aligned} u &= x^2 + 1 \\ du &= 2x dx \\ x^2 &= u - 1 \end{aligned}}$$

$$\begin{aligned} \frac{1}{2} \int \frac{x^2}{\sqrt{x^2+1}} 2x dx &= \frac{1}{2} \int \frac{u-1}{\sqrt{u}} du \\ &= \frac{1}{2} \int (\sqrt{u} - \frac{1}{\sqrt{u}}) du = \frac{1}{2} \left(\frac{2}{3} u^{3/2} - 2u^{1/2} \right) + C \\ &= \frac{1}{2} \left(\frac{2}{3} (x^2+1)^{3/2} - 2(x^2+1)^{1/2} \right) + C \end{aligned}$$

$$(4) \int_0^{\pi} \sec^2(t/4) dt =$$

$$\boxed{\begin{aligned} u &= t/4 \\ du &= \frac{1}{4} dt \\ t=0 &\rightarrow u=0 \\ t=\pi &\rightarrow u=\pi/4 \end{aligned}}$$

$$\begin{aligned} \frac{\pi}{4} \int_0^{\pi/4} \sec^2(u) du &= 4 \left[\tan(u) \right]_0^{\pi/4} = 4(1-0) \\ &= 4 \end{aligned}$$

$$(5) \int_0^4 \frac{x dx}{\sqrt{1+2x}} =$$

$$\boxed{\begin{aligned} u &= 1+2x \\ du &= 2 dx \\ x=0 &\rightarrow u=1 \\ x=4 &\rightarrow u=9 \end{aligned}}$$

~~$$\frac{1}{2} \int_1^9 \frac{u-1}{\sqrt{u}} du = \frac{1}{2} \int_1^9 (u^{1/2} - \frac{1}{\sqrt{u}}) du$$~~

$$\begin{aligned} \frac{1}{2} \int_1^9 \frac{u-1}{\sqrt{u}} du &= \frac{1}{2} \int_1^9 (u^{1/2} - \frac{1}{\sqrt{u}}) du \\ &= \frac{1}{2} \left[\frac{2}{3} u^{3/2} - 2u^{1/2} \right]_1^9 \\ &= \frac{1}{2} \left[(18 - 6) - \left(\frac{2}{3} - 2 \right) \right] \\ &= \frac{1}{2} \left[12 + \frac{4}{3} \right] = \frac{10}{3} \end{aligned}$$