

MAT 106
Quiz #18
November 11, 2004

Name: Key

You may not use your notes. Please show all of your work. An answer without justification will receive little credit.

- (1) Find the area bounded by the curves $y = 5x - x^2$ and $y = x$.

Limits of Integration:

$$\begin{aligned} 5x - x^2 &= x \\ \Leftrightarrow x^2 - 4x &= 0 \\ \Leftrightarrow x(x-4) &= 0 \\ \Leftrightarrow x &= 0, 4 \end{aligned}$$

$$(4-x)x$$

$$4x - x^2 = [5x - x^2] - (x)$$

$(-\infty, 0)$	$(0, 4)$	$(4, \infty)$
-	+	-

$$\begin{aligned} A &= \int_0^4 [(5x - x^2) - x] dx = \int_0^4 (4x - x^2) dx = \left[2x^2 - \frac{1}{3}x^3 \right]_0^4 \\ &= \left(32 - \frac{64}{3} \right) - (0) = \frac{96 - 64}{3} = \frac{32}{3} \end{aligned}$$

- (2) Find the area bounded by the curves $y^2 - 4y = x$ and $y^2 - 2y = -x$.

Limits of integration:

$$\begin{aligned} y^2 - 4y &= 2y - y^2 \\ \Leftrightarrow 2y^2 - 6y &= 0 \\ \Leftrightarrow 2y(y-3) &= 0 \\ \Leftrightarrow y &= 0, 3 \end{aligned}$$

$$(2y - y^2) - (y^2 - 4y)$$

$$= 6y - 2y^2$$

$$= 2y(3 - y)$$

$(-\infty, 0)$	$(0, 3)$	$(3, \infty)$
-	+	-

$$\begin{aligned} A &= \int_0^3 (2y - y^2 - (y^2 - 4y)) dy = \int_0^3 (6y - 2y^2) dy = \left[3y^2 - \frac{2}{3}y^3 \right]_0^3 \\ &= (9 - 0) = 9. \end{aligned}$$