

# MTHSC 206 SECTION 13.1 –THREE DIMENSIONAL COORDINATE SYSTEMS

Kevin James

# THREE DIMENSIONAL COORDINATE SYSTEM

## GOAL

We wish to generalize the familiar  $xy$ -plane to three dimensions in order to model the three dimensional space we live in. In order to do this we need to introduce some new ideas.

# THREE DIMENSIONAL COORDINATE SYSTEM

## GOAL

We wish to generalize the familiar  $xy$ -plane to three dimensions in order to model the three dimensional space we live in. In order to do this we need to introduce some new ideas.

- We use the traditional  $x$ - and  $y$ -axes and add a third  $z$ -axis which is perpendicular to the  $xy$ -plane.

# THREE DIMENSIONAL COORDINATE SYSTEM

## GOAL

We wish to generalize the familiar  $xy$ -plane to three dimensions in order to model the three dimensional space we live in. In order to do this we need to introduce some new ideas.

- We use the traditional  $x$ - and  $y$ -axes and add a third  $z$ -axis which is perpendicular to the  $xy$ -plane.
- The positive direction along the  $z$ -axis will be determined by the so called *right-hand rule*.

# THREE DIMENSIONAL COORDINATE SYSTEM

## GOAL

We wish to generalize the familiar  $xy$ -plane to three dimensions in order to model the three dimensional space we live in. In order to do this we need to introduce some new ideas.

- We use the traditional  $x$ - and  $y$ -axes and add a third  $z$ -axis which is perpendicular to the  $xy$ -plane.
- The positive direction along the  $z$ -axis will be determined by the so called *right-hand rule*.
- To each point  $P$  in space we associate a 3-tuple  $(a, b, c)$ .

# THREE DIMENSIONAL COORDINATE SYSTEM

## GOAL

We wish to generalize the familiar  $xy$ -plane to three dimensions in order to model the three dimensional space we live in. In order to do this we need to introduce some new ideas.

- We use the traditional  $x$ - and  $y$ -axes and add a third  $z$ -axis which is perpendicular to the  $xy$ -plane.
- The positive direction along the  $z$ -axis will be determined by the so called *right-hand rule*.
- To each point  $P$  in space we associate a 3-tuple  $(a, b, c)$ .
- To arrive at the point  $P$ , we travel  $a$  units along the  $x$ -axis,  $b$  units in the direction parallel to the  $y$ -axis and  $c$  units in the direction parallel to the  $z$ -axis.

## NOTE

There three planes determined by any two of the 3 axes are called the  $xy$ -plane, the  $xz$ -plane and the  $yz$ -plane.

## NOTE

There three planes determined by any two of the 3 axes are called the  $xy$ -plane, the  $xz$ -plane and the  $yz$ -plane.

## EXERCISE

What solution spaces are determined by the following?

- $z = 1$ .
- $x = y$ .
- $x = y = z$ .
- $x = 2, y = 1, z = 3$ .

### DEFINITION (DISTANCE FORMULA)

The distance  $|P_1P_2|$  between two points  $P_1(x_1, y_1, z_1)$  and  $P_2(x_2, y_2, z_2)$  is given by

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

### DEFINITION (DISTANCE FORMULA)

The distance  $|P_1P_2|$  between two points  $P_1(x_1, y_1, z_1)$  and  $P_2(x_2, y_2, z_2)$  is given by

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

### NOTE

This formula follows from the Pythagorean theorem.

### DEFINITION (DISTANCE FORMULA)

The distance  $|P_1P_2|$  between two points  $P_1(x_1, y_1, z_1)$  and  $P_2(x_2, y_2, z_2)$  is given by

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

### NOTE

This formula follows from the Pythagorean theorem.

### EXERCISE

Find the distance between  $(2, 1, 3)$  and  $(1, -1, 5)$ .

## EXERCISE

Give an equation whose solution set is the points on the surface of the sphere centered at the point  $(h, k, l)$  and whose radius is  $r$ .

### EXERCISE

Give an equation whose solution set is the points on the surface of the sphere centered at the point  $(h, k, l)$  and whose radius is  $r$ .

### EXERCISE

What region of  $\mathbb{R}^3$  is determined by the following inequalities.

$$1 \leq x^2 + y^2 + z^2 \leq 4, \quad z \leq 0$$