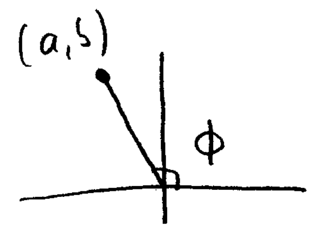
MTHSC 208, Fall 2011

Week 6 summary:

- Simple harmonic motion: $x'' = -\omega^2 x$ has solution

$$x(t) = a\cos\omega t + b\sin\omega t$$
$$= A\cos\left(\omega\left(t - \frac{\phi}{\omega}\right)\right), \quad A = \sqrt{a^2+b^2}, \quad \phi = \text{"}\arctan b/a\text{"}$$



- General harmonic motion: $mx'' + 2cx' + m\omega_0^2 x = f(t)$

Newton's 2^{nd} damping force spring force driving force

* with damping: ($c \neq 0$, say $f(t)=0$): Roots $r_{1,2} = -c \pm \sqrt{c^2 - \omega_0^2}$

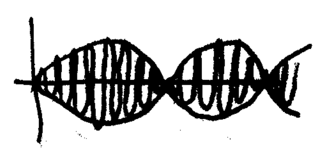
case 1: $c > \omega_0$ overdamped $x(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

case 2: $c < \omega_0$ underdamped $x(t) = e^{-ct}(a\cos\omega_0 t + b\sin\omega_0 t)$

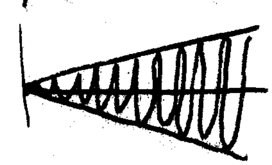
case 3: $c = \omega_0$ critically damped $x(t) = C_1 e^{rt} + C_2 t e^{rt}$

* forced harmonic motion ($f(t) \neq 0$). e.g., $x'' + \omega_0^2 x = A\cos\omega t$, $x(0) = x'(0) = 0$

case 1: $\omega \neq \omega_0$: $x(t) = \left(\frac{A}{2\delta\bar{\omega}}\sin\delta t\right)\sin\bar{\omega}t$



case 2: $\omega = \omega_0$: $x(t) = \left(\frac{A}{2\omega_0}t\right)\sin\omega_0 t$



- Basic linear algebra:

A system of linear equations
$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

can be written as $\boxed{A\vec{x} + \vec{b}}$, or
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}.$$