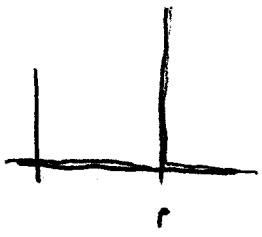
MTHSC 208: Fall 2011

Week 12 summary:

- Delta function: $\delta_p(t) = \begin{cases} \text{"}\infty\text{"} & t = p \\ 0 & t \neq p \end{cases} = \lim_{\varepsilon \to 0} \delta_p^{\varepsilon}(t)$



  * not really a function
  * is the "derivative" of $H_p(t)$
  * $\int_{-\infty}^{\infty} \delta_p(t)\, dt = 1$, $\int_{-\infty}^{\infty} \delta_p(t)\, F(t)\, dt = f(p)$
  * $\mathcal{L}(\delta_0(t)) = 1$, $\mathcal{L}(\delta_p(t)) = e^{-sp}$
  * Models a "unit impulse force."

- For $Per_{2\pi} = \{2\pi\text{-periodic functions}\}$, define

  $\langle f(x), g(x) \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, g(x)\, dx$. This allows us to compute the Fourier series: $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$

  $a_n = \langle f(x), \cos nx \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$

  $b_n = \langle f(x), \sin nx \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\, dx$