# MthSc 208: Fall 2011

## Week 13 summary:

- **Even functions:** $f(x) = f(-x)$

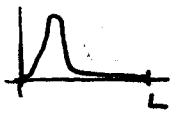
  * Symmetric about y-axis
  * Fourier series contains only cosines
  * $\int_{-L}^{L} f(x)\,dx = 2\int_{0}^{L} f(x)\,dx$

  **Odd functions:** $f(x) = -f(-x)$

  * Symmetric about origin
  * Fourier series contains only sines
  * $\int_{-L}^{L} f(x)\,dx = 0$

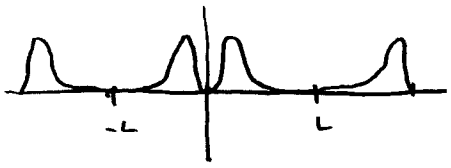
- **Fourier cosine & sine series:**

  Start with a function $f(x)$ on $[0, L]$, e.g.,

  * **Fourier cosine series** is the Fourier series of the **even** extension.

  e.g.,

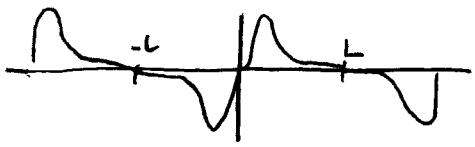
  
  

  $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$

  $$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

  * **Fourier sine series** is the Fourier series of the **odd** extension.

  e.g.,

  $$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

  $$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

- **Complex Fourier series:** $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx} = c_0 + \sum_{n=1}^{\infty}\left(c_n e^{inx} + c_{-n} e^{-inx}\right)$

  $$c_n = \langle f(x), e^{inx}\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,e^{-inx}\,dx \quad \text{(for } 2\pi\text{-periodic functions)}$$

  $$a_n = c_n + c_{-n} \qquad c_n = \frac{a_n - ib_n}{2}$$

  $$b_n = i(c_n - c_{-n}) \qquad c_{-n} = \frac{a_n + ib_n}{2}$$