MTHSC 208: Fall 2011

Week 14 & 15 summary:

- Partial differential equations (PDE's): Equations involving a multivariate function and its partial derivatives.

- Heat equation: $u_t = c^2 u_{xx}$, $u(x,t)$ = temp. at pos. x, time t.

  * Boundary conditions: e.g., $u(0,t) = u(L,t) = 0$
  * Initial conditions: e.g., $u(x,0) = h(x)$

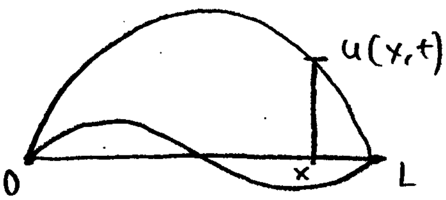
- Solving the heat equation: $u_t = c^2 u_{xx}$
  * Assume $u(x,t) = f(x)\,g(t)$. Compute u_t, u_{xx}, "zero-boundary conditions"
  * Plug back in and separate variables, set equal to λ.
  * Solve ODE's for $g(t)$ and $f(x)$, and determine λ.
  * Get a sol'n $u_n(x,t) = f_n(x)\,g_n(t)$ for each n.
  * Gen'l sol'n is $u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$ (superposition)
  * Plug in $t=0$ & use initial condition (may require finding a Fourier sine or cosine series.

- Boundary conditions for the heat equation.
  * Dirichlet: specify the value, e.g., $u(0,t) = T_1$, $u(L,t) = T_2$
  * von Neumann: specify the derivative, e.g., $u_x(0,t) = 0$, $u_x(L,t) = 0$.

    This represents insulated endpoints.

If boundary conditions are non-zero: $u(x,t) = u_h(x,t) + u_{ss}(x)$.

- <u>Wave equation</u>: $u_{tt} = c^2 u_{xx}$



Boundary conditions: $u(0,t) = u(L,t) = 0$

Initial conditions: $u(x,0) = h_1(x)$ "initial position"

$u_t(x,0) = h_2(x)$ "initial velocity"

Main difference: $g(t) = a\cos(cnt) + b\sin(cnt)$ instead of $Ae^{-c^2n^2t}$.