MTHSC 208: Fall 2011

Week 16 summary:

- PDEs in n dimensions
  - * Heat equation: $u_t = \nabla^2 u$
  - * Wave equation: $u_{tt} = \nabla^2 u$
  - * Laplace's equation: $\nabla^2 u = 0$

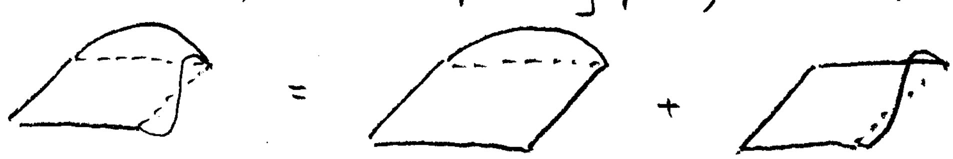
  where $\nabla^2 u = \dfrac{\partial^2 u}{\partial x_1^2} + \dots + \dfrac{\partial^2 u}{\partial x_n^2}$, the "Laplacian" of u.

- Harmonic functions: $\nabla^2 u = 0$.

Harmonic functions $\nabla^2 u = 0$	$\xleftrightarrow{\text{same}}$	steady-state sol'ns to the heat eq'n	$\xleftrightarrow{\text{same}}$	"Flat" functions; soap bubble or plastic wrap solutions

- Solving Laplace's equation in 2D: $u_{xx} + u_{yy} = 0$.

  Separate variables, do it piece-by-piece, use superposition.



- Solving the 2D heat equation: $u_t = c^2(u_{xx} + u_{yy})$.
  - * Assume $u(x,y,t) = f(x,y)\, g(t)$, plug in & separate variables.
  - * Get the Helmholtz eq'n for f: $\nabla^2 f = \lambda f$, $\lambda = -(n^2+m^2)$.
  - * General sol'n: $u(x,y,t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} u_{nm}(x,y,t)$
  - * Use initial condition: Plug in $t=0$ and equate coefficients.

If boundary conditions are non-zero: $u(x,y,t) = u_h(x,y,t) + u_{ss}(x,y)$.