MthSc 208: Differential Equations (Fall 2011)

In-class Worksheet 4c: Systems of differential equations (complex eigenvalues)

NAME:

Consider the system of differential equations: $\begin{cases} x_1' = -0.5x_1 + x_2, & x_1(0) = 0 \\ x_2' = -x_1 - 0.5x_2, & x_2(0) = 1 \end{cases}$

1. Write this in matrix form, $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b}$.

2. Given that the eigenvalues of $\mathbf{A}$ are $\lambda_1 = -\frac{1}{2} + i$ and $\lambda_2 = -\frac{1}{2} - i$, with associated eigenvectors $\mathbf{v}_1 = (1, i)$ and $\mathbf{v}_2 = (1, -i)$, write the general solution to $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

3. Use Euler's formula ($e^{it} = \cos t + i \sin t$) to write a solution (e.g., $\mathbf{x}_1(t)$) as a sum of its real and imaginary parts: $\mathbf{x}(t) = \mathbf{u}(t) + i\mathbf{w}(t)$.

4. Write the general solution as a linear combination of *real-valued* functions: $\mathbf{x}(t) = C_1\mathbf{u}(t) + C_2\mathbf{w}(t)$.

5. Find the particular solution satisfying the initial condition.

6. Sketch the phase portrait of the system. Also sketch the particular solution satisfying the initial condition.