1. Two fair dice are rolled. What is the conditional probability that at least one lands on 6 given that the dice land on different numbers?

2. A recent college graduate is planning to take the first three actuarial examinations in the coming summer. She will take the first exam in June. If she passes that exam, then she will take the second exam in July, and if she also passes that one, she will take the third exam in September. If she fails an exam, then she is not allowed to take any others. The probability that she passes the first exam is 0.9. If she passes the first exam, then the conditional probability that she passes the second one is 0.8, and if she passes that exam, then the conditional probability that she passes the third exam is 0.7.

   (a) What is the probability that she passes all three exams?

   (b) Given that she did not pass all three exams, what is the conditional probability that she failed the second exam?

3. Suppose that an ordinary deck of 52 cards is randomly divided into 4 hands of 13 cards each. We are interested in determining p, the probability that each hand has an ace. Let E_i be the event that the i^{th} hand has exactly one ace. Determine $p = P(E_1 E_2 E_3 E_4)$.

4. An ectopic pregnancy is twice as likely to develop when the pregnant woman is a smoker as it is when she is a nonsmoker. If 32 percent of women of childbearing age are smokers, what percentage of women having ectopic pregnancies are smokers?

5. A total of 48 percent of the women and 37 percent of the men that took a certain "quit smoking" class remained nonsmokers for at least one year after completing the class. These people then attended a success party at the end of a year. If 62% of the original class was male,

   (a) What percentage of those attending the party were women?

   (b) What percentage of the original class attended the party?

6. There are 15 tennis balls in a box, of which 9 have not previously been used. Three of the balls are randomly chosen, played with, and then returned to the box. Later, another 3 alls are randomly chosen from the box. Find the probability that none of these balls has ever been used.

7. A deck of cards is shuffled and then divided into two halves of 26 cards each. A card is drawn from one of the halves; it turns out to be an ace. The ace is then placed in the second half-deck. The half is then shuffled, and a card is drawn from it. Compute the probability that this drawn card is an ace. *Hint*: Condition on whether or not the interchanged card is selected.

8. Suppose that we want to generate the outcome of the flip of a fair coin, but that all we have at our disposal is a biased coin which lands on heads with some unknown probability p that need not be equal to $\frac{1}{2}$. Consider the following procedure for accomplishing our task:

   1. Flip the coin.

2. Flip the coin again.
3. If both flips are the same, return to step 1.
4. Let the result of the last flip be the result of the experiment.

(a) Show that the result is equally likely to be either heads or tails.

(b) Could we use a simpler procedure that continues to flip the coin until the last two flips are different, and then lets the result be the outcome of the final flip?

9. A town council of 7 members contains a steering committee of size 3. New ideas for legislation go first to the steering committee and then on to the council as a whole if at least 2 of the 3 committee members approve the legislation. Once at the full council, the legislation requires a majority vote to pass. Consider a new piece of legislation, and suppose that each town council member will approve it, independently, with probability p. What is the probability that a given steering committee memember's vote is decisive in the sense that if that person's vote were reverse, then the final fate of the legislation would be reversed? What is the corresponding probability for a given council member not on the steering committee?

10. Suppose that each child born to a couple is equally likely to be a boy or girl. For a couple having 5 children, compute the probabilities of the following events:

(a) All children are of the same sex.

(b) The 3 eldest are boys and the others girls.

(c) Exactly 3 are boys.

(d) The 2 oldest are girls.

(e) There is at least 1 girl.