

1. Two balls are chosen randomly from an urn containing 8 white, 4 black, and 2 orange balls. Suppose that we win \$2 for each black ball selected and we lose \$1 for each white ball selected. Let X denote our winnings. What are the possible values of X , and what are the probabilities associated with each value?
2. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. What are the possible values of X ?
3. A salesman has scheduled two appointments to sell encyclopedias. His first appointment will lead to a sale with probability 0.3, and his second will lead independently to a sale with probability 0.6. Any sale made is equally likely to be either for the deluxe model, which costs \$1000, or the standard model, which costs \$500. Determine the probability mass function of X , the total dollar value of all sales.
4. Suppose that the distribution function of X is given by

$$F(b) = \begin{cases} 0 & b < 0 \\ \frac{b}{4} & 0 \leq b < 1 \\ \frac{1}{2} + \frac{b-1}{4} & 1 \leq b < 2 \\ \frac{11}{12} & 2 \leq b < 3 \\ 1 & 3 \leq b \end{cases}$$

- (a) Find $P\{X = i\}$, for $i = 1, 2, 3$.
 - (b) Find $P\{\frac{1}{2} < X < \frac{3}{2}\}$.
5. A gambling book recommends the following “winning strategy” for the game of roulette: Bet \$1 on red. If red appears (which has probability $\frac{18}{38}$), then take the \$1 profit and quit. If red does not appear and you lose this bet (which has probability $\frac{20}{38}$ of occurring), make additional \$1 bets on red on each of the next two spins of the roulette wheel and then quit. Let X denote your winnings when you quit.
 - (a) Find $P\{X > 0\}$.
 - (b) Are you convinced that the strategy is indeed a “winning” strategy? Explain your answer!
 - (c) Find $E[X]$.
 6. You have \$1000, and a certain commodity presently sells for \$2 per ounce. Suppose that after one week the commodity will sell for either \$1 or \$4 an ounce, with these two possibilities being equally likely.
 - (a) If your objective is to maximize the expected amount of money that you possess at the end of the week, what strategy should you employ?
 - (b) If your objective is to maximize the expected amount of the commodity that you possess at the end of the week, what strategy should you employ?

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7. A newsboy purchases papers at 10 cents and sells them at 15 cents. However, he is not allowed to return unsold papers. If his daily demand is a binomial random variable with $n = 10$, $p = \frac{1}{3}$, approximately how many papers should he purchase so as to maximize his expected profit?
 8. If $E[X] = 1$ and $\text{Var}(X) = 5$, find
 - (a) $E[(2 + X)^2]$;
 - (b) $\text{Var}(4 + 3X)$.
 9. On a multiple-choice exam with 3 possible answers for each of the 5 questions, what is the probability that a student will get 4 or more correct answers just by guessing?
 10. Suppose that a biased coin that lands on heads with probability p is flipped 10 times. Given that a total of 6 heads results, find the conditional probability that the first 3 outcomes are
 - (a) H, T, T ;
 - (b) H, T, H .