1. Let f be an orientation-reversing element of $\text{Isom}(\mathbf{H}^2)$ such that $\tau(f) = 0$.
   (a) Prove that f is conjugate to the map $z \mapsto -\bar{z}$.
   (b) Prove that if r is a hyperbolic reflection, then $\tau(r) = 0$.
   (c) Prove that all hyperbolic reflections are conjugate.

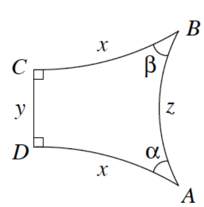
2. Let f be an orientation-reversing element of $\text{Isom}(\mathbb{R}^2)$ such that $\tau(f) < 0$.
   (a) Prove that f is conjugate to the map $z \mapsto -a\bar{z}$ for some $a > 1$.
   (b) Let g be another orientation-reversing element of $\text{Isom}(\mathbf{H}^2)$ such that $\tau(g) < 0$. Prove that f and g are conjugate if and only if $\tau(f) = \tau(g)$.
   (c) Prove that f is the product of a hyperbolic reflection f and an isometry t of hyperbolic type such that $tr = rt$. Analogous to the Euclidean case, isometries of this type are called *hyperbolic glide-reflections*.

3. Consider a hyperbolic isosceles right triangle with hypotenuse of length y and other sides of length x, and consider y as a function of x.
   (a) Find a formula relating x and y, and using implicit differentiation, find a formula for y' as a function of x and y.
   (b) Prove that $\sqrt{2} < y' < 2$ for $x > 0$, that $\lim_{x\to 0} y'(x) = \sqrt{2}$, and $\lim_{x\to\infty} y'(x) = 2$.

4. This problem proves our third "Thin Triangles Theorem" for hyperbolic triangles.
   (a) Consider a hyperbolic isoceles right triangle with two sides of length x, and the hypotenuse of length $y(x)$. Using the results of the previous problem and differential calculus, prove that $y'' > 0$ for $x > 0$.
   (b) Now, consider a fixed hyperbolic isoceles right triangle with two sides of length b and hypotenuse of length a. For $x \leq b$, let $y(x)$ be as before. Find and prove a relationship between a/b and y/x.
   (c) How does your answer in Part (b) compare to the Euclidean case?

   In the field of geometric group theory, this property of $\mathbf{H}^2$ is called CAT(0) (pronounced "cat-zero"), and spaces with this property are said to be *non-positively curved*. A similar but more general statements holds for arbitrary (non-isoceles) hyperbolic triangles.

5. Consider the hyperbolic quadrilateral $ABCD$ with $h(C, D) = y$, $h(A, B) = z$, $h(A, D) = h(B, C) = x$, and right angles at vertices C and D, as shown below.



(a) Prove that $\alpha = \beta < \pi/2$, and that $z > y$.

(b) Now, let $f \in \text{Isom}(\mathbf{H}^2)$ be an isometry of hyperbolic type. Prove that there is a uniue geodesic q such that $f(q) = q$, setwise, and prove that q is the set of minimal motion of f.

6. Consider the vertical lines L_1 and L_2 in $\mathbf{H}^2$ that have equations $x = -1/2$ and $x = 1/2$, respectively, and let $\omega = (-\frac{1}{2}, \frac{\sqrt{3}}{2}) \in \mathbf{H}^2$.

   (a) Find a parabolic $p \in \text{Isom}(\mathbf{H}^2)$ such that $p(L_1) = L_2$.

   (b) Suppose $f \in \text{Isom}(\mathbf{H}^2)$ is an orientation-preserving isometry such that the order of f is 3. Prove that $\tau(f) = 1$.

   (c) Find an orientation-preserving $f \in \text{Isom}(\mathbf{H}^2)$ such that $f(\omega) = \omega$ and the order of f is 3. [*Hint*: Normalize f to make $\det f = 1$, and justify why you can do this.]