1. Let $f, g \in \text{Isom}(\mathbf{H}^2)$ be orientation-preserving, and suppose that f is parabolic. Prove that either there exists $x \in \hat{\mathbb{R}}$ such that f and g both fix x, or $f^n g$ is an isometry of hyperbolic type for some value of n. [*Hint*: Use conjugation and "standard position."]

2. For $n \in \mathbb{N}$, a group element f has *order* n if n is the smallest positive integer such that $f^n = 1$. If such an n exists, then f has *finite order*, otherwise it has *infinite order*.

   (a) Prove that conjugate elements in any group have the same order.

   (b) Let $f \in \text{Isom}(\mathbf{H}^2)$ be orientation-preserving. Prove that if f has finite order, then f is elliptic.

   (c) Suppose that $f \in \text{Isom}(\mathbf{H}^2)$ is elliptic of order 5. Find all possible value of $\tau(f)$. [*Hint*: Use conjugation and basic properties of 2×2 rotation matrices.]

3. Show that the connected sum of two projective planes is homeomorphic to the Klein bottle. You may use "cut-and-paste" arguments.

4. Recall that a side-paring takes a polygon and pairs its sides in either an orientation-preserving or an orientation-reversing manner.

   (a) Find all possible ways to side-pair a square, up to rotational symmetry of the square.

   (b) For each side-pairing you found in Part (a), compute the Euler characteristic of the resulting surface S, and identify S has either a sphere, torus, Klein bottle, or projective plane.

5. Let S be the surface obtained by identifying opposite sides of an 18-gon in an orientation-preserving manner, that is, with the defining relation $a_1 a_2 \cdots a_9 a_1^{-1} a_2^{-1} \cdots a_9^{-1}$. Find with proof either a connected sum of tori T^2 and/or projective planes P^2 that is homeomorphic to S.

6. Prove that $P^2 \# T^2$ is homeomorphic to $P^2 \# P^2 \# P^2$.