

1. Consider a model of a structured population with matrix $P = \begin{pmatrix} .3 & 2 \\ .4 & 0 \end{pmatrix}$ (called a *Leslie matrix*):
 - (a) By thinking about the biological meaning of each entry in this matrix, do you think it describes a growing or declining population. Would you guess the population size would change rapidly or slowly? Explain your reasoning.
 - (b) Compute the eigenvalues and eigenvectors of the model.
 - (c) What is the intrinsic growth rate?
 - (d) Express the initial vector $\mathbf{x}_0 = (5, 5)$ as a sum of the eigenvectors.
 - (e) Use your answer in the previous part to give a formula for the population vector \mathbf{x}_t .
 - (f) What is the steady-state population, $\lim_{t \rightarrow \infty} \mathbf{x}_t$?

2. Repeat the last problem for the *Usher matrix* $P = \begin{pmatrix} 0 & 0 & 73 \\ .04 & 0 & 0 \\ 0 & .39 & .65 \end{pmatrix}$ with $\mathbf{x}_0 = (100, 10, 1)$.

3. A model given in (Cullen, 1985), based on data collected in (Nellis and Keith, 1976), describes a certain coyote population. The population is stratified in three classes: pup, yearling, and adult, and the matrix

$$P = \begin{pmatrix} .11 & .15 & .15 \\ .3 & 0 & 0 \\ 0 & .6 & .6 \end{pmatrix}$$

describes changes over a time step of 1 year.

- (a) Explain what each entry in this matrix is saying about the population. Be careful in explaining the $P_{1,1} = .11$ entry.
 - (b) Find the growth rate and steady-state distribution of this population.
 - (c) Will the population grow or decline? Quickly or slowly?
4. In class, we saw that the model

$$\begin{aligned} P_{t+1} &= P_t(1 + 1.3(1 - P_t)) - .5P_tQ_t \\ Q_{t+1} &= .3Q_t + 1.6P_tQ_t \end{aligned}$$

has a steady-state equilibrium that is approached through oscillations. Because the discrete logistic model $P_{t+1} = P_t(1 + 1.3(1 - P_t))$ on which it is based has $r = 1.3$, we know that it alone would produce underdamped dynamics (=damped oscillations) rather than the overdamped dynamics that arise when $r < 1$. Thus, it is not clear whether the oscillations in the model above are inherent to the model or, simply due to $r > 1$.

By using the MATLAB program `twopop` with a number of values of r less than 1.3 in the predator-prey model, see if you can find a value of $r < 1$ that yields oscillations in the predator-prey model. If so, can you find a value of r that yields no oscillations, and where is the “threshold” between these two dynamical regimes?

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5. Imagine a predator–prey interaction in which a certain number of the prey population cannot be eaten because of a refuge in their environment that the predator cannot enter.
- (a) Give an real-life example two populations that might exhibit this feature.
 - (b) Why might interaction terms like $-s(P - w)Q$ and $v(P - w)Q$ be reasonable in the modeling equation?
 - (c) What is the meaning of w ? Would you expect $w > P$ or $w < P$ to be more reasonable?