Read: Chapter 1: Mechanisms of gene regulation: Boolean network models of the lactose operon in *Escherichia coli*, by R. Robeva, B. Kirkwood, and R. Davis, pages 1–35.

Consider a Boolean model of the *lac* operon, based on five variables: mRNA (M), β-galactosidase (B), lac permease (P), intracellular lactose (L), and allolactose (A), and the following transition functions:

$$\begin{aligned} f_M &= A \\ f_B &= M \\ f_A &= A \vee (L \wedge B) \\ f_L &= P \vee (L \wedge \overline{B}) \\ f_P &= M \end{aligned}$$

This model does not have any parameters – it assumes that extracellular lactose is always available and extracellular glucose is always unavailable, and thus it is only able to describe the behavior of the system under the conditions.

1. Sketch the wiring diagram for this model.
2. Sketch the state space for this model. Feel free to use the *Discrete Visualizer of Dynamics* (DVD) software: `dvd.vbi.vt.edu`, but let $x_1 = M$, $x_2 = B$, $x_3 = A$, $x_4 = L$, and $x_5 = P$.
3. There are 3 fixed points: $(0,0,0,0,0)$, $(1,1,1,1,1)$, and $(0,0,0,1,0)$. Give a biological interpretation of the first two.
4. Explain why the fixed point $(0,0,0,1,0)$ does not make sense biologically.
5. Since the dynamics do not accurately reflect the behavior of the biological system it is meant to model, something is wrong. For each function, decide if it accurately reflects the underlying biology and/or the model assumptions.
6. Propose a modification of the transition functions aimed at eliminating the biologically infeasible fixed point. Give the rationale for your modification and specify the biological mechanism or model assumptions that justify the change.
7. Draw the wiring diagram and state space of your modified model. Use the DVD software.
8. Analyze your model. How many fixed points are there? Do they all correspond to biologically realistic situations? Note that there should be no limit cycles of size $k \geq 2$. (Why?)