Read: Chapter 2: Bistability in the lactose operon of *Escherichia coli*: A comparison of differential equation and Boolean network models, by R. Robeva, and N. Yildirim. pages 37–57.

1. Consider the reactions where two substrates S and T compete for binding to an enzyme E to produce two different products P and Q:

$$E + S \underset{p_2}{\overset{p_1}{\rightleftharpoons}} ES \xrightarrow{p_3} P + E$$
$$E + T \underset{q_2}{\overset{q_1}{\rightleftharpoons}} ET \xrightarrow{q_3} Q + E$$

   (a) Assuming that each reaction follows the Michaelis-Menten kinetics, derive rate equations for P and Q in this system. That is, determine $d[P]/dt$ and $d[Q]/dt$.

   (b) Explain the effects of the competition occuring.

2. The Hill equation is an approximation for multi-molecule binding and it assumes simultaneous binding of n-moecules of a substrate S to the enzyme E. Suppose that two molecules of the substrate S are undergoing a reaction with an enzyme in an ordered manner as follows:

$$E + S \underset{k_2}{\overset{k_1}{\rightleftharpoons}} ES + S \underset{k_4}{\overset{k_3}{\rightleftharpoons}} ES_2 \xrightarrow{k_5} P + E.$$

   (a) Derive a rate equation under the steady state assumption and compare it with the Hill equation with Hill coefficient $n = 2$:

$$\frac{d[P]}{dt} = \frac{V_{\max}[S]^2}{K_m + [S]^2}.$$

   (b) When do these two equations become roughly the same?