1. Consider a model of a structured population with matrix $P = \begin{pmatrix} .3 & 2 \\ .4 & 0 \end{pmatrix}$ (called a *Leslie matrix*):
   (a) By thinking about the biological meaning of each entry in this matrix, do you think it describes a growing or declining population. Would you guess the population size would change rapidly or slowly? Explain your reasoning.
   (b) Compute the eigenvalues and eigenvalues of the model.
   (c) What is the intrinsic growth rate?
   (d) Express the initial vector $\boldsymbol{x}_0 = (5, 5)$ as a sum of the eigenvectors.
   (e) Use your answer in the previous part to give a formula for the population vector $\boldsymbol{x}_t$.
   (f) What is the steady-state population, $\lim_{t\to\infty} \boldsymbol{x}_t$?

2. Repeat the last problem for the *Usher matrix* $P = \begin{pmatrix} 0 & 0 & 73 \\ .04 & 0 & 0 \\ 0 & .39 & .65 \end{pmatrix}$ with $\boldsymbol{x}_0 = (100, 10, 1)$.

3. A model given in (Cullen, 1985), based on data collected in (Nellis and Keith, 1976), describes a certain coyote population. The population is stratified in three classes: pup, yearling, and adult, and the matrix

$$P = \begin{pmatrix} .11 & .15 & .15 \\ .3 & 0 & 0 \\ 0 & .6 & .6 \end{pmatrix}$$

   describes changes over a time step of 1 year.
   (a) Explain what each entry in this matrix is saying about the population. Be careful in explaining the $P_{1,1} = .11$ entry.
   (b) Find the growth rate and steady-state distribution of this population.
   (c) Will the population grow or decline? Quickly or slowly?

4. In class, we saw that the model

$$\begin{aligned} P_{t+1} &= P_t(1 + 1.3(1 - P_t)) - .5P_tQ_t \\ Q_{t+1} &= .3Q_t + 1.6P_tQ_t \end{aligned}$$

   has a steady-state equilibrium that is approached through oscillations. Because the discrete logistic model $P_{t+1} = P_t(1 + 1.3(1 - P_t))$ on which it is based has $r = 1.3$, we know that it alone would produced underdamped dynamics (=damped oscillations) rather than the overdamped dynamics that arise when $r < 1$. Thus, it is not clear whether the oscillations in the model above are inherient to the model or, simply due to $r > 1$.

   By using the MATLAB program `twopop` with a number of values of r less than 1.3 in the predator–prey model, see if you can find a value or $r < 1$ that yields oscillations in the predator–prey model. If so, can you find a value of r that yields no oscillations, and where is the "threshold" between these two dynamical regimes?

5. Imagine a predator–prey interaction in which a certain number of the prey population cannot be eaten because of a refuge in their environment that the predator cannot enter.
   (a) Give an real-life example two populations that might exhibit this feature.
   (b) Why might interaction terms like $-s(P-w)Q$ and $v(P-w)Q$ be reasonable in the modeling equation?
   (c) What is the meaning of w? Would you expect $w > P$ or $w < P$ to be more reasonable?