Read: Chapter 2.3: Modeling biochemical reactions with differential equations, by R. Robeva, and N. Yildirim. pages 40–46.

1. Consider the reactions where two substrates S and T compete for binding to an enzyme E to produce two different products P and Q:

$$E + S \stackrel{p_1}{\underset{p_2}{\longrightarrow}} ES \stackrel{p_3}{\longrightarrow} P + E$$
$$E + T \stackrel{q_1}{\underset{q_2}{\longrightarrow}} ET \stackrel{q_3}{\longrightarrow} Q + E$$

- (a) Assuming that each reaction follows the Michaelis-Menten kinetics, derive rate equations for P and Q in this system. That is, determine d[P]/dt and d[Q]/dt.
- (b) Explain the effects of the competition occuring.
- 2. The Hill equation is an approximation for multi-molecule binding and it assumes simultaneous binding of n-molecules of a substrate S to the enzyme E. Suppose that two molecules of the substrate S are undergoing a reaction with an enzyme in an ordered manner as follows:

$$E + S \stackrel{k_1}{\underset{k_2}{\longleftrightarrow}} ES + S \stackrel{k_3}{\underset{k_4}{\longleftrightarrow}} ES_2 \stackrel{k_5}{\longrightarrow} P + E.$$

(a) Derive a rate equation under the steady state assumption and compare it with the Hill equation with Hill coefficient n = 2:

$$\frac{d[P]}{dt} = \frac{V_{\max}[S]^2}{K_m + [S]^2} \,.$$

(b) When do these two equations become roughly the same?