MATH 3110 - Fall 2015

Homework 6

Due: Thursday October 15

Questions. Chapter 4.2 of Strang **(total of 20 marks)**

1. Let $S = \langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \rangle$ be a line of $\mathbb{R}^3$. Project the vectors $b_1 = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$ and $b_2 = \begin{pmatrix} -5 \\ -7 \\ -3 \end{pmatrix}$ onto S. (4 marks)

2. Let $S = \langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \rangle$ bea line in $\mathbb{R}^3$. Project the following points onto S: (6 marks)

$$b_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},\ b_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \text{ and } b_3 = \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix}.$$

3. Consider the subset $S = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \mid x_1 - x_2 - 2x_3 = 0 \right\} \subseteq \mathbb{R}^3$. (8 marks)
   (a) Prove that S is a subspace of $\mathbb{R}^3$.
   (b) Find the dimension of S and give a basis of it.
   (c) Consider the basis to be the columns of a matrix A_1 such that $S = C(A_1)$. Compute the projection matrix P_1 for S.
   (d) Find another basis for S and compute the projection matrix P_2. Notice that $P_1 = P_2$, meaning that the projection matrix does not depend on the choice of the basis.

4. Show that if P is a projection matrix, then $I - P$ is a projection matrix. (2 marks)
   (Show that $(I - P)^T = I - P$ and $(I - P)^2 = I - P$)