Homework 9

QUESTION 1.   *Chapter 5 of Strang*   *(total of 20 marks)*

1. Using only properties from Section 5.1, compute the determinant of the following matrices with respect   (6 marks)
   to the parameter λ. For which values of λ are the matrices singular?

   (a) $\begin{pmatrix} \lambda & 1 & 2 \\ \lambda & \lambda & 3 \\ \lambda & \lambda & \lambda \end{pmatrix}$.

   (b) $\begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix} - \lambda I$ where I is the 2×2 identity matrix.

2. Show using the cofactor formular that if   (4 marks)

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}, B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix} \text{ and } D = \begin{pmatrix} d_{1,1} & d_{1,2} \\ d_{2,1} & d_{2,2} \end{pmatrix}$$

   then

$$\begin{vmatrix} A & B \\ 0 & D \end{vmatrix} = |A| \cdot |D|.$$

3. Compute the following determinant of the matrix   (3 marks)

$$\begin{pmatrix} a & 0 & 0 & 0 & 0 & b \\ 0 & a & 0 & 0 & b & 0 \\ 0 & 0 & a & b & 0 & 0 \\ 0 & 0 & c & d & 0 & 0 \\ 0 & c & 0 & 0 & d & 0 \\ c & 0 & 0 & 0 & 0 & d \end{pmatrix}$$

   (Hint: permute first rows and columns in order to obtain a "better" matrix.)

4. Solve the equation   (4 marks)

$$\det \begin{pmatrix} 1 & x & x^2 & x^3 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \end{pmatrix} = 0.$$

5. Show that the following determinant is equal to 0:   (3 marks)

$$\begin{vmatrix} 0 & 0 & 0 & a & b \\ 0 & 0 & 0 & c & d \\ 0 & 0 & 0 & e & f \\ p & q & r & s & t \\ v & w & x & y & z \end{vmatrix}$$