Homework 3

Due: Thursday September 21

ALWAYS MOTIVATE THE ANSWERS!

QUESTION 1. *Chapter 3.1 of Strang*   *(total of 30 marks)*

1. Which of the following subsets of $\mathbb{R}^3$ are actually subspaces? (Motivate the answers)   (8 marks)

   (a) The plane of vectors $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ with $b_2 = b_3$.

   (b) The plane of vectors with $b_1 = b_3 = 1$.

   (c) The vectors with $b_1 b_2 = 0$.

   (d) All linear combinations of $v = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $w = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}$.

2. Show that the following subsets V of $\mathbb{R}^3$ are not subspaces, meaning find a counterexample to one   (6 marks)
   of the main properties.

   (a) $V = P \setminus L = \{x \in \mathbb{R}^3 \mid x \in P \text{ and } x \notin L\}$, meaning all the elements of P that are
   not elements of L, where P is a plane passing through 0 and L is a line passing through 0 in $\mathbb{R}^3$.

   (b) $V = \{x \in \mathbb{R}^3 \mid \|x\| = 1\}$ where $\|x\|$ represents the length of a vector $x \in \mathbb{R}^3$.
   (Hint: V is a sphere of radius 1.)

   (c) $V = \{x \in \mathbb{R}^3 \mid \|x\| \leq 1\}$.
   (Hint: V is a ball of radius 1.)

3. Consider the following matrices   (7 marks)

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 3 & 1 & 0 \end{pmatrix}.$$

   (a) Show that both matrices are singular?

   (b) Describe their column spaces, $C(A)$ and $C(B)$? Which space are they?

   (c) Show that the columns of B are elements of $C(A)$.

   (d) Show that the columns of A are elements of $C(B)$.

4. Construct a 3×3 matrix whose column space contains only the vector $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.   (3 marks)

5. Construct a 3×3 matrix whose column space contains vectors $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and not $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ (3 marks)

6. Construct a 3×3 matrix whose column space is a line.   (3 marks)