MATH 3110 - Fall 2018
Homework 9

Due: Thursday November 1

QUESTION 1. *Chapter 5.1 of Strang* *(total of 24 marks)*

Solve the following questions by only using the properties of determinants. State the properties you use.

1. Using only properties from Section 5.1, compute the determinant of the following matrices with respect to the parameter λ. For which values of λ are the matrices singular? (8 marks)

   (a) $\begin{pmatrix} \lambda & 1 & 2 \\ \lambda & \lambda & 3 \\ \lambda & \lambda & \lambda \end{pmatrix}$.

   (b) $\begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix} - \lambda I$ where I is the 2×2 identity matrix.

2. Compute the following determinant of the matrix (4 marks)

$$\begin{pmatrix} a & 0 & 0 & 0 & 0 & b \\ 0 & a & 0 & 0 & b & 0 \\ 0 & 0 & a & b & 0 & 0 \\ 0 & 0 & c & d & 0 & 0 \\ 0 & c & 0 & 0 & d & 0 \\ c & 0 & 0 & 0 & 0 & d \end{pmatrix}$$

   (Hint: permute first rows and columns in order to obtain a "better" matrix.)

3. Solve the equation (6 marks)

$$\det \begin{pmatrix} 1 & x & x^2 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix} = 0.$$

   (Hint: Use a combination of Gaussian elimination and linearity.)

4. Show that the following determinant is equal to 0: (6 marks)

$$\begin{vmatrix} 0 & 0 & 0 & a & b \\ 0 & 0 & 0 & c & d \\ 0 & 0 & 0 & e & f \\ p & q & r & s & t \\ v & w & x & y & z \end{vmatrix}$$

   (Hint: How do you know that the rows are linearly dependent?)

QUESTION 2. *Chapter 5.2 of Strang* *(total of 6 marks)*

1. Show using the cofactor formular that if (6 marks)

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}, B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix} \text{ and } D = \begin{pmatrix} d_{1,1} & d_{1,2} \\ d_{2,1} & d_{2,2} \end{pmatrix}$$

   then

$$\begin{vmatrix} A & B \\ 0 & D \end{vmatrix} = |A| \cdot |D|.$$