MATH 3110 - Spring 2014

Homework 2

Due: Feb. 6th (Thursday)

Question 1. Chapter 2 of Strang **(total of 15 marks)**

1. If P_1 and P_2 are permutation matrices, so is P_1P_2. Give examples of: (2 marks)
   - matrices P_1, P_2 such that $P_1P_2 \neq P_2P_1$, and
   - matrices P_3, P_4 such the $P_3P_4 = P_4P_3$ when the neither of the matrices is the identity matrix.

2. Find the $PA = LU$ factorizations of the following matrices: (4 marks)

$$A = \begin{pmatrix} 0 & -2 & 2 \\ 2 & 0 & 2 \\ 4 & 2 & 4 \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} 0 & 0 & 3 \\ 1 & 1 & 9 \\ 2 & 1 & 3 \end{pmatrix}.$$

3. If A and B are symmetric matrices, which of the following matrices is symmetric? (Motivate the answer) (5 marks)

   (a) $A^2 - B^2$ (b) $(A+B)(A-B)$ (c) ABA (d) $ABAB$ (e) A^{-1}

4. (a) Let $A = \begin{pmatrix} 9 & -4 & -2 \\ -9 & 6 & -3 \\ 10 & -3 & 9 \end{pmatrix}$. Find matrices B, C such that A=B+C with (2 marks)

$$B = B^T \text{ (symmetric), and } C = -C^T \text{ (anti-symmetric).}$$

   (b) Find formulas for B and C involving A and A^T. We want $A = B + C$, $B = B^T$ and $C = -C^T$. (2 marks)

Question 2. Chapter 3 of Strang **(total of 15 marks)**

1. Which of the following subsets of $\mathbb{R}^3$ are actually subspaces? (Motivate the answers) (6 marks)

   (a) The plane of vectors $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ with $b_2 = b_3$.

   (b) The plane of vectors with $b_1 = b_3 = 1$.

   (c) The vectors with $b_1b_2 = 0$.

   (d) All linear combinations of $v = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}$.

   (e) All vectors that satisfy $b_1 + b_2 + b_3 = 0$.

   (f) all vectors with $b_1 \geq b_2 \geq b_3$.

2. The set $\mathbb{M}$ of all 2×2 matrices is a vector space. Describe the smallest subspace of $\mathbb{M}$ that contains (3 marks)

   (a) $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

3. (a) Describe the column space of $A = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$. Which subspace is it? (2 marks)

   (b) Construct a 3×3 matrix whose column space contains vectors $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and not $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ (2 marks)

   (c) Construct a 3×3 matrix whose column space is a line. (2 marks)