MATH 3110 - Spring 2014

Homework 3

Due: Feb. 13th (Thursday)

Questions. Chapter 3 of Strang **(total of 20 marks)**

1. Prove or disprove the following statements about the vector space $\mathbb{M}$ of $n \times n$ matrices (4 marks)
   (a) The set of upper triangular matrices is a subspace of $\mathbb{M}$.
   (b) The set of anti-symmetric matrices, i.e., all A such that $A = -A^T$, is a subspace of $\mathbb{M}$.
   (c) The set of invertible matrices, i.e. all A such that $\mathrm{rank}(A) = n$, is a subspace.
   (d) The set of singular matrices, i.e., all A such that $\mathrm{rank}(A) < n$, is a subspace.

2. Compute the row reduced echelon form of the following matrices (3 marks)

$$A = \begin{pmatrix} 1 & 2 & 2 & 3 & 9 \\ 3 & 6 & 1 & 4 & 7 \\ 0 & 0 & 1 & 1 & 4 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & 3 \\ 4 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

3. Find the special solutions of the nullspace of the following matrices (3 marks)

$$A = \begin{pmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

4. (a) Construct a matrix A (if possible) such that $N(A)$ contains all multiples of $\begin{pmatrix} 0 & 1 & 2 & 1 \end{pmatrix}^T$. (2 marks)
   (b) Construct a matrix A (if possible) such that (2 marks)
   - $C(A)$ contains $\begin{pmatrix} 2 & 4 & 2 \end{pmatrix}^T$ and $\begin{pmatrix} 1 & -1 & 0 \end{pmatrix}^T$, and
   - $N(A)$ contains $\begin{pmatrix} 1 & 2 & 2 \end{pmatrix}^T$.

   (c) Construct a matrix A (if possible) such that (2 marks)
   - $C(A)$ contains $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^T$ and $\begin{pmatrix} 1 & 1 & 0 \end{pmatrix}^T$, and
   - $N(A)$ contains $\begin{pmatrix} 0 & 1 & 1 \end{pmatrix}^T$ and $\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^T$.

5. Find the set of solutions of the following set of linear equations (4 marks)

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 3x_4 = 4 \\ x_1 + 2x_2 + 3x_3 + x_4 = 2 \\ x_1 + 2x_2 + 3x_3 + 2x_4 = 3 \end{cases}$$