MATH 3110 - Spring 2014

Homework 7

Due: Mar. 13th (Thursday)

Question 1. Chapter 4.2 of Strang **(total of 14 marks)**

1. Let $S = \langle \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}^T \rangle$ be a line of $\mathbb{R}^3$. Project the vectors $\begin{pmatrix} 5 & 7 & 3 \end{pmatrix}^T$ and $\begin{pmatrix} -5 & -7 & -3 \end{pmatrix}^T$ onto S. (3 marks)

2. Consider the subset $S \subseteq \mathbb{R}^4$ defined by the equation $x - y - 2z = 0$. (6 marks)

   (a) Find the dimension of S and give a basis of it.

   (b) Consider the basis to be the columns of a matrix A_1 such that $S = C(A_1)$. Compute the projection matrix P_1 for S.

   (c) Find another basis for S and compute the projection matrix P_2. Notice that $P_1 = P_2$, meaning that the projection matrix does not depend on the choice of the basis.

3. Show that if P is a projection matrix, then $I - P$ is a projection matrix. (2 marks)

4. Let $S \subset \mathbb{R}^n$. Show that for every vector $v \in \mathbb{R}^n$ there exist two vectors $v_S \in S$ and $v_{S^\perp} \in S^\perp$ such that $v = v_S + v_{S^\perp}$. (3marks)

Question 2. Chapter 4.3 of Strang **(total of 6 marks)**

1. Consider the four data points $(t_i, b_i) = (0, 0),\ (1, 8),\ (3, 8)$ and $(4, 20)$. (6 marks)
   Find the best fitting line $b = C + Dt$ between the points.