

MATH 3110 - Spring 2014

Homework 9

Due: Apr. 3rd (Thursday)

Question. Chapter 5.2 and 5.3 of Strang

(total of 20 marks)

1. Show using the cofactor formula that if

(3 marks)

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}, B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix} \text{ and } D = \begin{pmatrix} d_{1,1} & d_{1,2} \\ d_{2,1} & d_{2,2} \end{pmatrix}$$

then

$$\begin{vmatrix} A & B \\ 0 & D \end{vmatrix} = |A| \cdot |D|.$$

2. Prove that the following determinant is equal to 0:

(4 marks)

$$\begin{vmatrix} 0 & 0 & 0 & a & b \\ 0 & 0 & 0 & c & d \\ 0 & 0 & 0 & e & f \\ p & q & r & s & t \\ v & w & x & y & z \end{vmatrix}$$

3. Compute the determinants of the matrices

(3 marks)

$$\begin{pmatrix} 13247 & 13347 \\ 28469 & 28569 \end{pmatrix} \text{ and } \begin{pmatrix} 246 & 427 & 327 \\ 1014 & 543 & 443 \\ -342 & 721 & 621 \end{pmatrix}$$

4. Compute the following determinant of the matrix

(3 marks)

$$\begin{pmatrix} a & 0 & 0 & 0 & 0 & b \\ 0 & a & 0 & 0 & b & 0 \\ 0 & 0 & a & b & 0 & 0 \\ 0 & 0 & c & d & 0 & 0 \\ 0 & c & 0 & 0 & d & 0 \\ c & 0 & 0 & 0 & 0 & d \end{pmatrix}$$

5. Solve the equation

(4 marks)

$$\begin{pmatrix} 1 & x & x^2 & x^3 \\ 1 & a_1 & a_1^2 & a_1^3 \\ 1 & a_2 & a_2^2 & a_2^3 \\ 1 & a_3 & a_3^2 & a_3^3 \end{pmatrix} = 0$$

when a_1, a_2 and a_3 are all different.

6. Prove that if all entries of A and A^{-1} are integers, then $\det(A)$ is either 1 or -1 .

(3 marks)