MATH 3110 - Spring 2014

Homework 10

Due: Apr. 10th (Thursday)

Question. Chapter 6.1 and 6.2 of Strang **(total of 20 marks)**

1. Compute the eigenvalues and eigenvectors of the following matrices (6 marks)

   (a) $A_1 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$ (b) $A_2 = A_1^{-1}$ (c) $A_3 = A_1^2 + 3I$

2. Prove that is A is an invertible matrix and λ is an eigenvalue of A, then λ^{-1} is an eigenvalue of A^{-1}. (3 marks)

3. Prove that A is a diagonal matrix if and only if the standard basis vectors are all eigenvectors of A. (3 marks)

4. Diagonalize matrix $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ by finding the matrices S and Λ. (4 marks)

5. Diagonalize A and compute $S\Lambda^k S^{-1}$ to prove this formula for A^k (4 marks)

$$A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \quad \text{and} \quad A^k = \frac{1}{2}\begin{pmatrix} 1+3^k & 1-3^k \\ 1-3^k & 1+3^k \end{pmatrix}$$

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