

Clemson Analysis Seminar Archive

Fall 2023

Electroconvection in Fluids

Speaker: Elie Abdo (University of California, Santa Barbara)

Date: September 13, 2023

Abstract: We consider an electroconvection model describing the time evolution of a surface charge density in a two-dimensional incompressible fluid. The charge density evolves according to a nonlinear nonlocal drift-diffusion equation. The drift velocity obeys Navier-Stokes equations forced by nonlinear electrical forces driven by the charge density and time-independent body forces in the fluid. We address the global well-posedness, global regularity, and long-time dynamics of the deterministic model. In the presence of a regular additive noise, we study the existence and uniqueness of a smooth invariant measure for the Markov transition kernels associated with the model.

Analytic Wavefront Sets of Spherical Distributions on De Sitter Space

Speaker: Iswarya Sitiraju (Louisiana State University)

Date: September 20, 2023

Abstract: A De Sitter space dS , is a one sheeted hyperboloid with Lorentzian metric. For G the orthochronous Lorentzian group acting linearly on $(n + 1)$ Lorentzian space $\mathbb{R}^{1,n}$ and the subgroup H , the orthochronous Lorentzian group acting on $\mathbb{R}^{1,n-1}$, the space dS is homogeneous space G/H . A distribution u is said to be a spherical distribution if it is H -invariant eigendistribution of the Laplace-Beltrami operator on dS . The dimension of the space of spherical distribution is two. I will construct the basis for this space as follows: The spherical function on the space G/K , where K is orthogonal group $O_n(\mathbb{R})$, can be extended as sesquiholomorphic kernel on an open complex domain called as crown domain. The de Sitter space lies on the boundary of this crown domain. The boundary value of this kernel on dS turns out to be the spherical distribution. Further, I will characterize the analytic wavefront set of these distributions, which is the set of points where the distribution is not analytic and the direction in which the singularity occurs.

Compactness of Toeplitz operators with continuous symbols on pseudoconvex domains in \mathbb{C}^n

Speaker: Tomas Rodriguez (University of Toledo)

Date: September 27, 2023

Abstract: Let Ω be a bounded pseudoconvex domain in \mathbb{C}^n with Lipschitz boundary and ϕ be a continuous function on $\bar{\Omega}$. We show that the Toeplitz operator T_ϕ with symbol ϕ is compact on the weighted Bergman space if and only if ϕ vanishes on the boundary of Ω . We also show that compactness of the Toeplitz operator T_ϕ^q on $\bar{\partial}$ -closed (p, q) -forms for $0 \leq p \leq n$ and $q \geq 1$ is equivalent to $\phi = 0$ on $\bar{\Omega}$. This material comes from the presenter's PhD thesis written at University of Toledo under the supervision of Sonmez Sahutoglu.

A Remarkable Discovery of Bourgain and Brezis

Speaker: Daniel Spector (National Taiwan Normal University)

Date: October 6, 2023

Abstract: In their 2004 and 2007 papers, Jean Bourgain and Haim Brezis proved several stunning results concerning estimates for PDE in the L^1 setting. The goal of this talk is to present the past, present, and a possible future of these estimates.

The flow of polynomial roots under differentiation

Speaker: Changhui Tan (University of South Carolina)

Date: October 13, 2023

Abstract: How roots of a polynomial evolve under repeated differentiation is a classical question, which goes back to Gauss, Lucas, Marcel Riesz and many others. The dynamics can be described by a partial differential equation, formally derived by Stefan Steinerberger recently.

In this talk, I will explain the intriguing relationship between the roots of trigonometric polynomials under differentiation and Steinerberger's PDE. The PDE is an active scalar equation, which is closely related to many classical models in fluid dynamics. In particular, the flow of polynomial roots bears a strong resemblance to self-organized dynamics in the modeling of animal swarms. The celebrated flocking behavior is connected to the "crystallization" phenomenon on the polynomial root distribution. I will discuss joint work with Alexander Kiselev on global wellposedness and asymptotic behavior of the PDE, as well as its rigorous connections towards the flow of polynomial roots.

Determining solution uniqueness of nonlinear equations by continuous data assimilation

Speaker: Xuejian Li (Clemson University)

Date: October 27, 2023

Abstract: The solution of a nonlinear equation is generally not unique. In this talk, we address such non-uniqueness problems in the setting where data measurements or solution observations are available. The steady Navier-Stokes equations (a typical nonlinear partial differential equation) will be considered as a concrete example to present. We use a continuous data assimilation (CDA) method to incorporate the observations into the Navier-Stokes equations (NSE) and obtain a CDA-NSE system. We prove that with sufficient observations, the CDA-NSE system is well-posed even for large Reynolds number where multiple solutions exist.

Inverse hyperbolic problems without the compactness-uniqueness argument

Speaker: Shitao Liu (Clemson University)

Date: November 8, 2023

Abstract: In this talk I will introduce a new strategy to directly derive stability estimates for inverse hyperbolic problems by using Carleman estimates. In particular, this does not require getting the uniqueness result first or using any cut-off procedure.

Recovery of time-dependent coefficients in hyperbolic equations on conformally transversally anisotropic manifolds from partial data

Speaker: Boya Liu (North Carolina State University)

Date: November 15, 2023

Abstract: We discuss inverse problems of determining time-dependent coefficients appearing in the wave equation in a compact Riemannian manifold of dimension three or higher. More specifically, we are concerned with the case of conformally transversally anisotropic manifolds, or in other words, compact Riemannian manifolds with boundary conformally embedded in a product of the Euclidean line and a transversal manifold. With an additional assumption of the attenuated geodesic ray transform being injective on the transversal manifold, we prove that the knowledge of a certain partial Cauchy data set determines time-dependent coefficients of the wave equation uniquely on a space-time cylinder. We shall discuss two problems: (1) Recovery of a potential appearing in the wave equation, with the Dirichlet value, in the Cauchy data, measured on only part of the lateral boundary of the space-time cylinder. (2) Recovery of both a damping coefficient and a potential appearing in the wave equation, with the Dirichlet value measured on the whole lateral boundary. This talk is based on joint works with Teemu Saksala (NC State University) and Lili Yan (University of Minnesota).

The compactness of bilinear Calderón-Zygmund operators

Speaker: Anastasios Fragkos (Washington University in St. Louis)

Date: December 8, 2023

Abstract: The compactness of singular integral operators, aside from its intrinsic interest, has been used in the field of Elliptic PDE. Villaroya in 2014 characterized the compactness of Calderón-Zygmund operators in terms of a weak compactness condition, certain kernel estimates, and membership of the 0-th order paraproducts in $CMO(\mathbb{R}^d)$. Recently, Mitkovski and Stockdale reformulated Villaroya's result following a simpler approach. We prove a wavelet $T(1)$ type theorem for the compactness of bilinear Calderón-Zygmund operators. The backbone of our approach is a representation theorem for compact bilinear CZOs with wavelet forms that reflect the compact nature of the operator. This talk is based on joint work with Walton Green and Brett Wick.

Spring 2023

Classical multiplier theorems and their sharp variants

Speaker: Lenka Slavíková (Charles University)

Date: January 18, 2023

Abstract: The question of finding good sufficient conditions on a bounded function m guaranteeing the L^p -boundedness of the associated Fourier multiplier operator is a long-standing open problem in harmonic analysis. In this talk, I will recall the classical multiplier theorems of Hörmander and Marcinkiewicz and present their sharp variants in which the multiplier belongs to a certain fractional Sobolev space. The talk is based in part on a joint work with L. Grafakos and M. Mastyło.

Frames via Unilateral Iterations of Bounded Operators

Speaker: Victor Bailey (Georgia Institute of Technology)

Date: February 3, 2023

Abstract: Dynamical Sampling is, in a sense, a hypernym classifying the set of inverse problems arising from considering samples of a signal and its future states under the action of a bounded linear operator. Recent works in this area consider questions such as when can a given frame for a separable Hilbert Space, $\{f_k\}_{k \in I} \subset H$, be represented by iterations of an operator on a single vector and what are necessary and sufficient conditions for a system, $\{T^n \varphi\}_{n=0}^\infty \subset H$, to be a frame? In this talk, we will discuss the connection between frames given by iterations of a bounded operator and the theory of model spaces in the Hardy-Hilbert Space as well as necessary and sufficient conditions for a system generated by the orbit of a pair of commuting bounded operators to be a frame.

Muckenhoupt weights and the reverse Hölder inequality on metric measure spaces

Speaker: Emma-Karoliina Kurki (Aalto University)

Date: February 10, 2023

Abstract: It is well known that in Euclidean spaces the reverse Hölder inequality (RHI) implies the Muckenhoupt A_p condition. This statement remains true in reasonably general metric measure spaces, but the necessary condition appears to be unknown. I give an overview of what is known and not known about the RHI as a characterization of A_p weights. Furthermore, I discuss characterizations of the weak reverse Hölder inequality (WRHI), which is the nondoubling analogy of the RHI with an increasing support on the right-hand side. The weak case is based on a joint work with Carlos Mudarra.

Convergence of discrete non-linear Fourier transform via spectral problems for canonical systems

Speaker: Ashley Zhang (University of Wisconsin, Madison)

Date: February 20, 2023

Abstract: This talk will be about connections between spectral problems for canonical systems and non-linear Fourier transforms (NLFTs). Non-linear Fourier transform is closely connected to Dirac systems, which form a subclass of canonical systems of differential equations. This connection allows one to find analogs of results on inverse spectral problems for

canonical systems in the area of NLFT. In particular, NLFTs of discrete sequences, discussed in the lecture notes by Tao and Thiele, are related to spectral problems for periodic measures and the theory of orthogonal polynomials. I will start the talk with the basics of non-linear Fourier transforms, then connect NLFTs to canonical systems. Then I will present an explicit algorithm for inverse spectral problems developed by Makarov and Poltoratski for locally-finite periodic spectral measures, as well as an extension of their work to certain classes of non-periodic spectral measures. Finally I will return to NLFT and translate the results for inverse spectral problems to results for NLFT.

Improved spectral cluster estimates on compact manifolds

Speaker: Xiaoqi Huang (University of Maryland)

Date: February 24, 2023

Abstract: We will discuss the problem of trying to obtain improved eigenfunction / spectral projection estimates under geometric assumptions, such as the presence of negative sectional curvatures, which is based on the use of microlocal Keakeya-Nikodym estimates along with a combination of local and global harmonic analysis that further exploit geometric assumptions. Using a similar approach, we will also discuss some improved Strichartz estimates on compact manifolds with negative curvature. This is based on joint work with Chris Sogge and Matthew Blair.

Operator-free sparse domination

Speaker: Emiel Lorist (Delft University of Technology)

Date: March 3, 2023

Abstract: Sparse domination is a recent technique, allowing to estimate (in norm, pointwise or dually) many operators in harmonic analysis by simple, positive expressions. This technique has led to many new results in harmonic analysis over the past decade. In this talk I will discuss a sparse domination principle for an arbitrary family of functions $f(x, Q)$, where $x \in \mathbb{R}^n$ and Q is a cube in \mathbb{R}^n . When applied to operators, this result recovers various recent sparse domination results. In contrast to preceding results, our sparse domination principle can also be applied to non-operator objects, which allows one to use sparse techniques in new areas. I will discuss new applications of sparse domination to weighted Poincaré inequalities and to potential theory. This talk is based on joint work with Andrei Lerner and Sheldy Ombrosi.

Exponential frams and point configurations

Speaker: Alex Iosevich (University of Rochester)

Date: March 17, 2023

Abstract: We are going to discuss the existence and non-existence of exponential bases and frames corresponding to domains in Euclidean space. A particular emphasis is placed on connections with techniques involving finite point configurations in geometric measure theory.

Analytic continuation of Toeplitz operators and their spectral representation on the weighted Bergmann spaces

Speaker: Khalid Bdarneh (Louisiana State University)

Date: April 12, 2023

Abstract: Weighted Bergman spaces on the unit ball $A_\lambda^2(\mathbb{B}^n)$ are defined for the weights $\lambda > n$. In this talk we will discuss how the weighted Bergmann spaces can be extended to $\lambda > 0$, and we will present a characterization of commuting families of C^* -algebras in terms of restriction to multiplicity free representations. Moreover, we will describe the spectral representation of Toeplitz operators that acts on $A_\lambda^2(\mathbb{B}^n)$ by extending the restriction principal to the analytic continuation case for suitable maximal abelian subgroups of $\widetilde{SU}(n, 1)$. This is a joint work with Dr. Gestur Ólafsson.

Dense subsets of the Toeplitz algebra on the Fock space

Speaker: Vishwa Dewage (Clemson University)

Date: April 26, 2023

Abstract: We study the full Toeplitz algebra via convolutions of operators and the laplacian of the Berezin transform. We present new classes of operators that are dense in the Toeplitz algebra. We use these new classes of operators to discuss dense subsets of some commutative subalgebras of the Toeplitz algebra. This is a joint work with Mishko Mitkovski.

Fall 2022

Optimal control for suppression of singularity in chemotaxis

Speaker: Weiwei Hu (University of Georgia)

Date: September 26, 2022

Abstract: In this talk, we discuss the problem of optimal control design for suppression of singularity via flow advection in chemotaxis modeled by the Patlak-Keller-Segel (PKS) equations. For the system without advection, it can exhibit dramatic collapsing behavior. Specifically, if the initial condition is above a certain critical threshold the solution may blow up at a finite time by concentrating positive mass at a single point. In this work we will focus on the design of an optimal flow field to suppress such singularities. A rigorous theoretical framework will be addressed and numerical experiments will be presented to demonstrate the ideas.

Recover All Coefficients in Second-order Hyperbolic Equations from Finite Sets of Boundary Measurements

Speaker: Shitao Liu (Clemson University)

Date: October 5, 2022

Abstract: In this talk we consider the inverse hyperbolic problem of recovering all spatial dependent coefficients, which are the wave speed, the damping coefficient, potential coefficient and gradient coefficient, in a second-order hyperbolic equation defined on an open bounded domain with smooth enough boundary. We show that by appropriately selecting finite pairs of initial conditions we can uniquely and Lipschitz stably recover all those coefficients from the corresponding boundary measurements of their solutions. The proofs are based on sharp Carleman estimate, continuous observability inequality and regularity theory for general second-order hyperbolic equations. This work is joint with Antonio Pierrottet and Scott Scruggs.

Mean-Field Games for Scalable Computation and Diverse Applications

Speaker: Wuchen Li (University of South Carolina)

Date: October 21, 2022

Abstract: Mean field games (MFGs) study strategic decision-making in large populations where individual players interact via specific mean-field quantities. They have recently gained enormous popularity as powerful research tools with vast applications. For example, the Nash equilibrium of MFGs forms a pair of PDEs, which connects and extends variational optimal transport problems. This talk will present recent progress in this direction, focusing on computational MFG and engineering applications in robotics path planning, pandemics control, and Bayesian/AI sampling algorithms. This is based on joint work with the MURI team led by Stanley Osher (UCLA).

Bounds on some classical exponential Riesz basis

Speaker: Thibaud Alemany (Georgia Institute of Technology)

Date: October 28 2022

Abstract: We estimate the Riesz basis (RB) bounds obtained in Hruschev, Nikolskii and Pavlov's classical characterization of exponential RB. As an application, we improve previously known estimates of the RB bounds in some classical cases, such as RB obtained by an Avdonin type perturbation, or RB which are the zero-set of sine-type functions. This talk is based on joint work with S. Nitzan.

The Möbius disjointness conjecture of Sarnak for ergodic weights

Speaker: Nikos Frantzikinakis (University of Crete)

Date: November 11, 2022

Abstract: The Möbius function is a multiplicative function which encodes important information related to distributional properties of the prime numbers. It is widely believed that its non-zero values fluctuate between plus and minus one in such a random way that causes non-correlation with any "reasonable" sequence of complex numbers. One conjecture in this direction, formulated by Sarnak, states that the Möbius function does not correlate with any bounded deterministic sequence, meaning, any sequence that is produced by a continuous function evaluated along the orbit of a point in a zero entropy topological dynamical system. I will give an overview of the proof of a joint result of mine with Bernard Host from 2018, in which we proved the logarithmically averaged variant of this conjecture for a wide class of dynamical systems, which includes all uniquely ergodic ones. Our approach is to study structural properties of measure preserving systems naturally associated with the Möbius function. I will explain how these structural results are obtained using a combination of tools from ergodic theory and analytic number theory, and how we use them for our purposes.

Spring 2022

Necessary Conditions for Two Weight Weak Type Norm Inequalities for Multilinear Singular Integral Operators

Speaker: John-Oliver MacLellan (University of Alabama)

Date: January 28, 2022

Abstract: A central problem in harmonic analysis is to characterize the pairs of weights (u, v) so that a Calderón Zygmund operator maps $L^p(v) \rightarrow L^p(u)$. In this talk we will discuss necessary conditions for a multilinear Calderón Zygmund operator T to satisfy two weight weak type norm inequalities provided the kernel of T satisfies a weak non degeneracy condition. We generalize results from our recent paper, and earlier results from Lacey, Sawyer, and Uriarte-Tuero, and by Stein in the linear case. As an application of our techniques, we will show that in general a multilinear Calderón Zygmund does not satisfy a two-weight strong endpoint estimate.

The C^* -algebra generated by Toeplitz operators with quasi-radial symbol

Speaker: Vishwa Dewage (Louisiana State University)

Date: February 4, 2022

Abstract: In this talk we discuss Toeplitz operators with k -quasi-radial symbols acting on the Fock space $\mathcal{F}(\mathbb{C}^n)$. Toeplitz operators with k -quasi-radial symbols generate a commutative C^* -algebra that is isometrically isomorphic to $C_{b,u}(\mathbb{N}_0^k)$ of bounded functions on \mathbb{N}_0^k that are uniformly continuous with respect to the square root metric. In fact, the spectral functions (multi-sequences of eigenvalues) of these Toeplitz operators are dense in the space $C_{b,u}(\mathbb{N}_0^k)$. This talk is based on a joint work with Prof. Gestur Olafsson.

On the boundedness of oscillating singular integrals

Speaker: Duván Cardona Sánchez (Ghent University)

Date: February 11, 2022

Abstract: It was proved by Fefferman [1] and Fefferman and Stein [2], the weak $(1, 1)$ boundedness of oscillating singular integrals on \mathbb{R}^n , and the boundedness from the Hardy space H^1 into L^1 , respectively. The aim of this talk is to discuss the recent extension of these results in the Euclidean setting (in view of the paradigm introduced by Grafakos and Stockdale in [3]) and on Lie groups of polynomial growth. In view of the solution of the Rockland conjecture by Helffer and Nourrigat [4], and of the Hormander theorem of sums of squares [5], our criteria are presented in terms of the analysis of sub-Laplacians and of Rockland operators. This talk is based on my joint works [CR1,CR2] with Michael Ruzhansky on the subject.

Space-time sampling for the functions with bounded spectrum

Speaker: Ilia Zlotnikov (University of Stavanger)

Date: February 28, 2022

Abstract: Let $I = [a, b]$, $0 < a < b < \infty$ and let $\varphi_{uu \in I}$ be a collection of functions that satisfy some natural assumptions (e.g., one may think that $\varphi_u(x) = e^{-ux^2}$). Assume that Λ is a uniformly discrete set. We will discuss what assumptions should be imposed on the

set Λ to provide a stable reconstruction of the function (signal) f belonging to the Paley-Wiener space PW_σ^p or Bernstein space B_σ from the samples $\{f * \varphi_u(\lambda)\}_{\lambda \in \Lambda}$, i.e. to ensure the estimates

$$\|f\|_{L^\infty} \leq K \sup_{u \in I, \lambda \in \Lambda} |f * \varphi_u(\lambda)|, \quad f \in B_\sigma,$$

or

$$D_1 \|f\|_{L^p}^p \leq \sum_{\lambda \in \Lambda} \int_I |f * \varphi_u(\lambda)|^p du \leq D_2 \|f\|_{L^p}^p, \quad f \in PW_\sigma^p,$$

with constants K , D_1 and D_2 independent of f . This problem has several practical applications. For instance, we provide conditions for stable recovery of the initial data of the heat equation from given space–time samples.

References: [1] A. Ulanovskii, I. Zlotnikov, Reconstruction of bandlimited functions from space–time samples, *J. Funct. Anal.* 280(9), 108962 (2021).

[2] I. Zlotnikov, On planar sampling with Gaussian kernel in spaces of bandlimited functions, preprint: <http://arxiv.org/abs/2104.09573>

Characterizations of the Toeplitz algebra on the Fock space

Speaker: Raffael Hagger (Christian Albrechts University in Kiel)

Date: March 9, 2022

Abstract: Let μ_α denote the Gaussian measure on \mathbb{C}^n given by

$$d\mu_\alpha(z) = \left(\frac{\alpha}{\pi}\right)^n e^{-\alpha|z|^2} dz$$

and consider the corresponding L^p -space $L_\alpha^p := L^p(\mathbb{C}^n, \mu_{\alpha p/2})$ for $p \in (1, \infty)$. The closed subspace of entire functions in L_α^p is called the Segal–Bargmann or Fock space and denoted by F_α^p . The corresponding orthogonal projection $P_\alpha : L_\alpha^2 \rightarrow F_\alpha^2$ can be extended to a projection from L_α^p onto F_α^p . Of particular interest in this context are the Toeplitz operators $T_f : F_\alpha^p \rightarrow F_\alpha^p$, which are given by

$$T_f g := P_\alpha(fg)$$

for bounded symbols $f : \mathbb{C}^n \rightarrow \mathbb{C}$, and the Banach algebra generated by all Toeplitz operators with bounded symbol. For instance, a bounded linear operator on the Fock space is compact if and only if it is in the Toeplitz algebra and its so-called Berezin transform vanishes at infinity [1]. However, it is notoriously difficult to check directly whether a given operator is contained in the Toeplitz algebra. Recently, Fulsche [2] showed quite intriguingly that methods from quantum harmonic analysis can be used to study the Toeplitz algebra. In this talk I will present some direct consequences of Fulsche’s work including several characterizations of the Toeplitz algebra which provide better membership criteria [3].

On Asymptotic Moments of Patterned Random Matrices

Speaker: Tapesh Yadav (University of Florida)

Date: March 18, 2022

Abstract: For a sufficiently nice 2 dimensional shape, we define its approximating matrix (or patterned matrix) as a random matrix with iid entries arranged according to the given pattern. For large approximating matrices, we observe that the eigenvalues roughly follow an underlying distribution. This phenomenon is similar to the classical observation on Wigner matrices. We prove that the moments of such matrices converge asymptotically as the size increases and equals to the integral of a combinatorially-defined function which counts certain paths on a finite grid.

Quantitative estimates in the matrix weighted setting

Speaker: Israel Rivera-Ríos (University of Málaga)

Date: April 1, 2022

Abstract: In this talk we will provide an overview from the quantitative theory of Muckenhoupt weights to the quantitative matrix weights theory. The plan of the talk will consist in revisiting sparse domination ideas, which have been a fruitful field of development in the scalar setting, and providing some insight on how they have been adapted to the vector valued setting, making them useful to obtain quantitative weighted estimates in that setting.

Integral geometry problems in Lorentzian geometry

Speaker: Yiran Wang (Emory University)

Date: April 8, 2022

Abstract: We consider the light ray transform on Lorentzian manifolds, which concerns the integral of functions along light-like (or null) geodesics. The transform is related to the Radon transform or geodesic ray transform in the Riemannian setting. An outstanding question is what information regarding the function can be recovered from the light ray transform. In this talk, we discuss recent developments, including injectivity, stability results and microlocal properties of the transform. Also, we discuss the application to some inverse problems in cosmology. In particular, we will show how to recover space-time structures from the Cosmic Microwave Background (CMB).

Kohler-Jobin meets Ehrhard: the sharp lower bound for the Gaussian principal frequency while the Gaussian torsional rigidity is fixed, via rearrangements

Speaker: Orli Herscovici (Georgia Institute of Technology)

Date: April 15, 2022

Abstract: In this talk, we show an adaptation of the Kohler-Jobin rearrangement technique to the setting of the Gauss space. As a result, we present the Gaussian analogue of the Kohler-Jobin's resolution of a conjecture of Polya-Szego: when the Gaussian torsional rigidity of a (convex) domain is fixed, the Gaussian principal frequency is minimized for the half-space. At the core of this rearrangement technique is the idea of considering a "modified" torsional rigidity, with respect to a given function, and rearranging its layers to half-spaces, in a particular way; the Rayleigh quotient decreases with this procedure.

We emphasize that the analogy of the Gaussian case with the Lebesgue case is not to be expected here, as in addition to some soft symmetrization ideas, the argument relies on the properties of some special functions; the fact that this analogy does hold is somewhat of a miracle. Based on joint work with Galyna Livshyts.

Inversion, convexity, and realization problems in free noncommutative analysis

Speaker: Mark Mancuso (Lafayette College)

Date: April 25, 2022

Abstract: This talk will provide an overview of some central problems in free analysis related to noncommutative inversion, matrix and operator convexity, and obtaining realizations. Free analysis, or noncommutative function theory, finds its origin in J. L. Taylor's work on the functional calculus of several noncommuting operators. Loosely speaking, noncommutative function theory can be thought of as an analogue of several complex variables where the holomorphic functions act on domains consisting of either tuples of bounded operators on a Hilbert space or tuples of square complex matrices of all sizes. The simplest examples of noncommutative functions are given by free polynomials with complex coefficients; $p(X, Y) = XY^2 + XY - YX$ is a free polynomial and can be evaluated at pairs of matrices or operators. Well-known results along with recent progress will be shared throughout this talk.

Fall 2021

Alpha-area Siegel-Veech constants for branched cyclic covers

Speaker: Martin Schmoll (Clemson University)

Date: September 8, 2021

Abstract: We will present a relative formula for asymptotic quadratic growth rates of periodic cylinders for branched cyclic covers that seems to hold in more generality. We will talk about to which extent we can prove this formula and its inner structure. Background on flat surfaces is provided (mostly informally), as well as a brief introduction into the Siegel-Veech formula and its evaluation. An interesting feature of the formula for general surfaces of higher genus is that it is essentially the formula for branched torus covers. It is worth noting that the resulting formula has been predicted by computer experiments using an approach that does not involve the Siegel-Veech formula our proof relies on.

This is ongoing research together with David Auricino (Brooklyn College), Aaron Calderon (Yale) and Nick Salter (Notre Dame).

A completeness and cyclicity problem for a semigroup of weighted composition operator

Speaker: Chris Felder (Washington University in St. Louis)

Date: September 17, 2021

Abstract: This talk will be largely expository, following recent work of S. Waleed Noor. We will discuss a completeness problem in the classical Hardy space, along with a cyclicity problem for a semigroup of weighted composition operators. We will then talk about a least-squares approach to solving such problems.

On the John-Nirenberg constant of BMO^p , $0 < p < 1$

Speaker: Brandon Sweeting (University of Alabama)

Date: September 24, 2021

Abstract: We present sharp L^p lower bounds for logarithms of A_∞ weights as a means of estimating the John-Nirenberg constant of the space BMO^p , $0 < p < 1$. The corresponding Bellman function solves the homogeneous Monge-Ampère equation, but the geometry of the solution goes beyond established theory due to the lack of regularity in the boundary condition. This is joint work with Leonid Slavin.

Stabilization rates for the damped wave equation with polynomial and oscillatory damping

Speaker: Perry Kleinhenz (Michigan State University)

Date: September 29, 2021

Abstract: In this talk I will discuss energy decay of solutions of the Damped wave equation. After giving an overview of classical results I'll focus on the torus with damping that does not satisfy the geometric control condition. In this setup properties of the damping at the boundary of its support determine the decay rate, however a general sharp rate is not known.

I will discuss damping which is 0 on a strip and vanishes either like a polynomial x^b or an oscillating exponential $e^{-1/x} \sin^2(1/x)$. Polynomial damping produces decay of the

semigroup at exactly $t^{-(b+2)/(b+3)}$, while oscillating damping produces decay at least as fast as $t^{-4/5+\delta}$ for any $\delta > 0$. I will explain how these model cases are proved and how they direct further study of the general sharp rate.

An approach to universality using Weyl m -functions

Speaker: Milivoje Lukić (Rice University)

Date: October 8, 2021

Abstract: In this talk, I will present joint work with Benjamin Eichinger and Brian Simanek: a new approach to universality limits for orthogonal polynomials on the real line which is completely local and uses only the boundary behavior of the Weyl m -function at the point. We show that bulk universality of the Christoffel–Darboux kernel holds for any point where the imaginary part of the m -function has a positive finite nontangential limit. This approach is based on studying a matrix version of the Christoffel–Darboux kernel and the realization that bulk universality for this kernel at a point is equivalent to the fact that the corresponding m -function has normal limits at the same point. Our approach automatically applies to other self-adjoint systems with 2×2 transfer matrices such as continuum Schrödinger and Dirac operators. We also obtain analogous results for orthogonal polynomials on the unit circle.

Random Interpolating Sequences in the Polydisc and the Unit Ball

Speaker: Alberto Dayan (Norwegian University of Science and Technology)

Date: October 15, 2021

Abstract: A random sequence Z in the unit disc is determined by a sequence of deterministic radii and a sequence of i.i.d. random variables uniformly distributed on the unit circle. Chochran and Rudowicz found the 0 – 1 Kolmogorov law for Z to be interpolating, that is, the cut-off condition on the a-priori fixed radii in order for Z to be interpolating almost surely. In this talk, we will extend their work to random interpolating sequences for bounded analytic functions in the d -dimensional polydisc and for the Besov-Sobolev spaces on the unit ball. The case of the Besov-Sobolev spaces is more treatable, since such spaces have their interpolating sequences well understood and characterized in the deterministic setting. This is not the case for interpolating sequences in the polydisc: in this second case our necessary and sufficient conditions for almost sure interpolation do not coincide, and they are obtained by looking at the decay of related random Grammians off their diagonals.

This is a joint work with Brett Wick and Shengkun Wu

Weighted Inequalities on Spaces of Homogeneous Type

Speaker: Manasa Vempati (Georgia Institute of Technology)

Date: October 27, 2021

Abstract: We will discuss the two weight inequalities for Calderón-Zygmund operators and commutators. We work in the setting of spaces of homogeneous type defined in the sense of Coifman and Weiss. Subject to the pair of weights u and v satisfying a side condition, we will show a characterization for the boundedness of a Calderon-Zygmund operator T from $L^2(u)$ to $L^2(v)$ in terms of the A_2 condition and two testing conditions. We will also give the two weight quantitative estimates for the commutator of maximal functions and the maximal commutators with respect to the symbol in weighted BMO space on spaces of homogeneous

type.

A wavepacket approach to the random Schrodinger equation

Speaker: Felipe Hernandez (Stanford University)

Date: November 1, 2021

Abstract: I will discuss my work in progress on the Schrodinger equation perturbed by a random potential. This equation is a simple model for wave propagation in random environments. A key feature of solutions to this equation is a diffusion phenomenon, meaning that the mass of the wavefunctions weakly solves a heat equation. A previous work of Erdos, Salmhofer, and Yau in 2007 derived this heat equation using a complex diagrammatic expansion. I will explain in this talk how wavepacket decompositions can be used to give a more geometric understanding of the problem.

A strange limit of horocycle ergodic measures in a stratum of translation surfaces

Speaker: Jon Chaika (University of Utah)

Date: November 12, 2021

Abstract: The main result of this talk is that in the space of unit area translation surfaces with one cone point there is a weak-star limit of measures on periodic horocycles that is fully supported in the 7-dimensional space but gives positive measure to a 3-dimensional submanifold. As a consequence we obtain a non-genericity result for the horocycle flow in this space. I will define the terminology. This is joint work with Osama Khalil and John Smillie.

Pressure Function on Compact Symbolic Systems

Speaker: Tamara Kucherenko (CUNY)

Date: November 19, 2021

Abstract: We study the flexibility of the pressure function of a continuous potential (observable) with respect to a parameter regarded as the inverse temperature. The points of non-differentiability of this function are of particular interest in statistical physics, since they correspond to phase transitions. It is well known that the pressure function is convex, Lipschitz, and has an asymptote at infinity. We prove that in a setting of one-dimensional compact symbolic systems these are the only restrictions. We present a method to explicitly construct a continuous potential whose pressure function coincides with any prescribed convex Lipschitz asymptotically linear function starting at a given positive value of the parameter. In fact, we establish a multidimensional version of this result. As a consequence, we obtain that for a continuous observable the phase transitions can occur at a countable dense set of temperature values. We go further and show that one can vary the cardinality of the set of ergodic equilibrium states as a function of the parameter to be any number, finite or infinite.

Optimal Tomography with Local Data

Speaker: Francis Chung (University of Kentucky)

Date: November 29, 2021

Abstract: Optical tomography is the problem of reconstructing interior properties of an object from optical measurements at the boundary. The mathematical version of this prob-

lem is to reconstruct the coefficients of a PDE (in my case, a radiative transport equation) from measurements of the solutions at the boundary. In this talk I'll discuss an optical tomography problem with local data, in which measurements are restricted to a subset of the boundary. I plan to introduce the problem, discuss the wider context, and describe a recent result.

Spring 2021

Translational tilings: structure and decidability

Speaker: Rachel Greenfeld (University of California Los Angeles)

Date: January 13, 2021

Abstract: Let F be a finite subset of the d -dimensional integer lattice. We say that F is a translational tile of \mathbb{Z}^d if it is possible to cover \mathbb{Z}^d by translates of F with no overlaps. Given a finite subset F of \mathbb{Z}^d , could we determine whether F is a translational tile in finite time? Suppose that F does tile, what can be said about the structure of the tiling? A well known argument of Wang shows that these two questions are closely related. In the talk, I will introduce and demonstrate this relation and present some new results, joint with Terence Tao, on the rigidity of tiling structures in \mathbb{Z}^2 , and their applications to decidability.

Thin elastic plates involving fractional rotational forces: semigroups regularity and stability

Speaker: Louis Tebou (Florida International University)

Date: January 20, 2021

Abstract: In this talk, finite thin elastic plates involving fractional rotational forces are considered. Using resolvent estimates, new regularity and stability results for the underlying semigroups are established. First, I examine a thermoelastic plate with fractional rotational forces, and prove Gevrey regularity, as well as exponential stability of the associated semigroup. Afterward, I analyze a mechanically damped plate, where the rotational forces and damping involve fractional powers of the Laplacian. For this latter model, it will be shown that the underlying semigroup is analytic, or of a certain Gevrey class, is exponentially or polynomially stable, depending on the relationship between the fractional exponents of the damping and rotational forces. The models considered are new and lie between the classical Euler-Bernoulli model and the Kirchhoff model for thin plates in either case. This is a joint-work with my colleagues Valentin Keyantuo (University of Puerto Rico, San Juan) and Mahamadi Warma (George Mason University)

Strongly Peaking Representations and Compressions of Operator Systems

Speaker: Benjamin Passer (United States Naval Academy)

Date: January 27, 2021

Abstract: (Joint work with Ken Davidson.) Just as a compact convex set is generated by its extreme points, it is known that an operator system is completely normed by its boundary representations. We analyze a special class of boundary representations in order to study operator systems which are presented in a smallest possible way. Our results extend recent work in the study of free spectrahedra and matrix convex sets.

Uncertainty principle in harmonic analysis and its applications

Speaker: Mishko Mitkovski (Clemson University)

Date: February 3, 2021

Abstract: I will present several new forms of the uncertainty principle. These new forms can be viewed as extensions/sharpening of some classical uncertainty principles, in a sense that we impose restrictions on the Fourier support and deduce sampling inequalities. Many

of our results were inspired by some control and damping problems for linear PDE's and I will try to present some of these applications. This is a collaborative work with W. Green, B. Jaye, and H. Li.

Data-Dependent Distances for Unsupervised and Active Learning

Speaker: James Murphy (Tufts University)

Date: February 10, 2021

Abstract: Approaches to unsupervised clustering and active learning with data-dependent distances are proposed. By considering metrics derived from data-driven graphs, robustness to noise and class geometry is achieved. The proposed algorithms enjoy theoretical guarantees on flexible data models, and also have quasilinear computational complexity in the number of data points. Connections will be made to geometric analysis and percolation theory. Applications to image processing and biological networks will be shown, demonstrating state-of-the-art empirical performance.

Weighted inequalities for Haar multipliers

Speaker: Claire Huang (Washington University in St. Louis)

Date: February 19, 2021

Abstract: A measure on \mathbb{R}^n is called “dyadic doubling” if the measure ratio of any dyadic parent and its dyadic child is uniformly bounded. This property enables a stopping time argument presented by Katz and Pereyra in their 1998 survey on the L^p boundedness of Haar multipliers. Highly inspired by these two authors’ work, this presentation focuses on extending their results to weighted spaces. In particular, we are interested in the L^p boundedness of Haar multipliers with respect to weights in dyadic Muckenhoupt classes, as they guarantee dyadic doubling, and the closely related weights in dyadic reverse Hölder classes.

Trace minmax functions and the radical Laguerre-Polya class

Speaker: James Pascoe (University of Florida)

Date: February 22, 2021

Abstract: We classify functions $f : (a, b) \rightarrow \mathbb{R}$ which satisfy the inequality $\operatorname{tr} f(A) + f(C) \geq \operatorname{tr} f(B) + f(D)$ when $A \leq B \leq C$ are self-adjoint matrices, $D = A + C - B$, the so-called trace minmax functions. (Here $A \leq B$ if $B - A$ is positive semidefinite, and f is evaluated via the functional calculus.) A function is trace minmax if and only if its derivative analytically continues to a self map of the upper half plane. The negative exponential of a trace minmax function $g = e^{-f}$ satisfies the inequality $\det g(A)g(C) \leq \det g(B)g(D)$ for A, B, C, D as above. We call such functions determinant isoperimetric. We show that determinant isoperimetric functions are in the “radical” of the the Laguerre-Polya class. We derive an integral representation for such functions which is essentially a continuous version of the Hadamard factorization for functions in the the Laguerre-Polya class. We apply our results to give some equivalent formulations of the Riemann hypothesis. Finally, we discuss some recent joint work with Kelly Bickel and Meredith Sargent on relaxations of the above framework, and an avenue for obtaining zero-free regions for the Riemann zeta function.

On dual molecules and convolution-dominated operators

Speaker: Jordy Timo van Velthoven (University of Vienna)

Date: March 3, 2021

Abstract: This talk considers coherent systems arising from unitary representations. It is shown that frames and Riesz sequences can be obtained whose dual systems form molecules, ensuring that their elements satisfy appropriate size estimates as if they were obtained one from the other by the group action. As a consequence, the canonical expansions extend to associated Banach spaces. The main tool is a local holomorphic calculus for convolution-dominated operators, valid for groups with possibly exponential volume growth.

Control and Inverse Problems for a Strongly Coupled Hyperbolic System of PDEs

Speaker: Jason Kurz (Clemson University)

Date: March 12, 2021

Abstract: In this presentation we consider an inverse and control problem for the Mindlin–Timoshenko plate system, which is a strongly coupled two dimensional system consisting of a wave equation and a system of isotropic elasticity, that arises in modeling plate vibrations especially at high frequencies and thicker plates. More precisely, we prove the global uniqueness of recovering the plate density from a single boundary measurement under appropriate geometrical assumptions. Secondly, we demonstrate the controllability of the system via an indirect control technique that proves a two-level indirect inverse observability estimate. Our approach for both problems relies on diagonalizing the principal part of the system and making it a system of wave-like equations.

Wavelet Representation of Singular Integral Operators

Speaker: Tyler Williams (Washington University in St. Louis)

Date: March 26, 2021

Abstract: We describe a new representation technique for one and multiple parameter singular integrals in terms of continuous model operators. Unlike the well established dyadic counterpart, our representation reflects the additional kernel smoothness of the operator being analyzed. Our representation formulas lead naturally to a new family of $T(1)$ theorems on weighted Sobolev spaces whose smoothness index is naturally related to kernel smoothness. I present the one parameter case, where we obtain the Sobolev space analogue of the A_2 theorem; that is, sharp dependence of the Sobolev norm of T on the weight characteristic is obtained in the full range of exponents. In the bi-parametric setting, where we obtain quantitative A_p estimates which are best known, and sharp in the range $\max p, p' \geq 3$. These estimates are beyond the reach of current dyadic methods.

Boundedness of commutators on weighted Hardy spaces

Speaker: Marie-Jose Kuffner (Johns Hopkins University)

Date: April 2, 2021

Abstract: It is known that boundedness of the commutator $[b, H]$ on weighted L^p -spaces for $1 < p < \infty$ is characterized by b being in a certain BMO space adapted to the given weights. In this talk, we present the case $p = 1$ and discuss the space that characterizes boundedness of $[b, H]$ on the weighted Hardy space $H^1(w)$ for certain A_p weights.

Inverse Problems for Nonlinear PDEs

Speaker: Ting Zhou (Northeastern University)

Date: April 7, 2021

Abstract: In this talk, I will demonstrate the higher order linearization approach to solve several inverse boundary value problems for nonlinear PDEs modeling nonlinear electromagnetic optics including nonlinear time-harmonic Maxwell's equations with Kerr-type and second harmonic generation nonlinearity. The problem will be reduced to solving for the coefficient functions from their integrals against multiple linear solutions. We will focus our discussion on different choices of linear solutions. A similar problem for nonlinear magnetic Schrodinger equation will be considered as a comparison.

Tiling the integers with translates of one tile: the Coven-Meyerowitz tiling conditions for three prime factors

Speaker: Itay Londner (University of British Columbia)

Date: April 12, 2021

Abstract: It is well known that if a finite set of integers A tiles the integers by translations, then the translation set must be periodic, so that the tiling is equivalent to a factorization $A + B = \mathbb{Z}_M$ of a finite cyclic group. Coven and Meyerowitz (1998) proved that when the tiling period M has at most two distinct prime factors, each of the sets A and B can be replaced by a highly ordered "standard" tiling complement. It is not known whether this behavior persists for all tilings with no restrictions on the number of prime factors of M .

In an ongoing collaborations with Izabella Laba, we proved that this is true when $M = (pqr)^2$. In my talk I will discuss this problem and introduce the main ingredients of the proof.

Extrapolation of compactness on weighted spaces

Speaker: Stefanos Lappas (University of Helsinki)

Date: April 21, 2021

Abstract: The extrapolation theorem of Rubio de Francia is one of the most powerful tools in the theory of weighted norm inequalities: it allows one to deduce an inequality (often but not necessarily: the boundedness of an operator) on all weighted L^p spaces with a range of p , by checking it just for one exponent p (but all relevant weights). My topic is an analogous method for extrapolation of compactness. In a relatively soft way, it recovers several recent results about compactness of operators on weighted spaces and also gives some new ones. This is a joint-work with Tuomas Hytönen.

Fall 2020

C-cyclical monotonicity in optimal transport and beyond

Speaker: Mathias Beiglböck (University of Vienna)

Date: September 9, 2020

Abstract: A fundamental idea in optimal transport is that the optimality of a transport plan is reflected by the geometry of its support set. Often this is key to understanding the transport problem. On the level of support sets, the relevant notion is c-cyclical monotonicity. The relevance of this concept for the theory of optimal transport has been fully recognized by Gangbo and McCann, based on earlier work of Knott and Smith and Ruschendorf among others. Since then it has been understood through the work of various authors that analogous "monotonicity principles" are also highly useful in a number of related areas. The goal of this talk is to provide an introduction to these ideas and to sketch some of the recent developments.

Weighted Estimates for the Bergman and Szego Projections on Strongly Pseudoconvex Domains with Near Minimal Smoothness

Speaker: Nathan Wagner (Washington University in St. Louis)

Date: September 23, 2020

Abstract: The Bergman and Szego projections are fundamental operators in complex analysis in one and several complex variables. Consequently, the mapping properties of these operators on L^p and other function spaces have been extensively studied. In this talk, we discuss some recent results for these operators on strongly pseudoconvex domains with near minimal smoothness. In particular, weighted L^p estimates are obtained, where the weight belongs to a suitable generalization of the Bekolle-Bonami or Muckenhoupt class. For these domains with less boundary regularity, we use an operator-theoretic technique that goes back to Kerzman and Stein. We also obtain weighted estimates for the endpoint $p = 1$, including weak-type $(1, 1)$ estimates. Here we use a modified version of singular-integral theory and a generalization of the Riesz-Kolmogorov characterization of precompact subsets of Lebesgue spaces. This talk is based on joint work with Brett Wick and Cody Stockdale.

What is a quantum Markov chain? A beginner's look into non-commutative probability

Speaker: Renato Feres (Washington University in St. Louis)

Date: October 7, 2020

Abstract: For a mathematician, basic quantum theory may be viewed as a generalized (non-commutative) probability theory. The current mathematical research in non-commutative probability and stochastic processes has been greatly influenced by scientific developments in quantum communication and the physics of open quantum systems. In this presentation, I wish to explore a few of the more fundamental ideas in (discrete time, Markovian) quantum stochastic processes, guided by a class of classical Markov chains obtained from elementary mechanical systems.

A Calderón-Zygmund decomposition for von Neumann algebra valued functions

Speaker: José Conde-Alonso Universidad Autónoma de Madrid

Date: October 14, 2020

Abstract: The classical Calderón-Zygmund decomposition is a fundamental tool that helps one study endpoint estimates near L^1 . In this talk, we shall study an extension of the decomposition to a particular operator valued setting where noncommutativity makes its appearance, allowing to get rid of the (usually necessary) UMD property of the Banach space where functions take values.

A two-dimensional inverse problem in magnetism

Speaker: Elodie Pozzi (St. Louis University)

Date: October 21, 2020

Abstract: Inverse problems have known a recent development in many fields like signal processing, medical imaging and more recently paleomagnetism. Broadly speaking, an inverse problem consists in reconstructing from a set of measurements the original source. We consider a two dimensional inverse problem in magnetism to estimate the net moment represented by the mean value of a function supported on an interval K of the real line from the partial knowledge of the magnetism on another interval S located on the parallel line to K at height h . We will see how this question can be rephrased using complex analysis, harmonic analysis and operator theory. To estimate the mean value, we will construct and solve a constrained approximation problem. This talk is based on a joint work with Juliette Leblond, INRIA, France.

Commutators and bounded mean oscillation

Speaker: Brett Wick (Washington University in St. Louis)

Date: October 28, 2020

Abstract: We will discuss some recent results about commutators of certain Calderón-Zygmund operators and BMO spaces and how these generate bounded operators on Lebesgue spaces. Results on the Heisenberg group, pseudoconvex domains with C^2 boundary, and other examples will be explained. This talk is based on joint collaborative work.

A different approach to endpoint weak-type estimates for Calderón-Zygmund operators

Speaker: Cody Stockdale (Clemson University)

Date: November 4, 2020

Abstract: The weak-type $(1, 1)$ estimate for Calderón-Zygmund operators is fundamental in harmonic analysis. We investigate weak-type inequalities for Calderón-Zygmund singular integral operators using the Calderón-Zygmund decomposition and ideas inspired by Nazarov, Treil, and Volberg. We discuss applications of these techniques in the Euclidean setting, in weighted settings, for multilinear operators, for operators with weakened smoothness assumptions, and in studying the dimensional dependence of the Riesz transforms.

Sharp trace inequalities involving differential operators

Speaker: Franz Gmeineder (University of Bonn)

Date: November 11, 2020

Abstract: Essentially by the non-availability of singular integrals on L^1 , one cannot estimate the L^1 -norms of the full k^{th} order gradients against those of k^{th} order differential

expressions in general. This is known as Ornstein's Non-Inequality. Over the past years, however, a theory has been developed that allows to estimate lower order quantities by the L^1 -norms of differential expressions in a sharp way. In this talk, which is joint work with Lars Diening (Bielefeld), we present sharp conditions on differential operators to retrieve the well-known boundary trace estimates from the Sobolev case.

Sampling, density and equidistribution

Speaker: José Luis Romero (University of Vienna)

Date: November 17, 2020

Abstract: The sampling problem concerns the reconstruction of every function within a given class from their values observed only at certain points (samples). A density theorem gives necessary or sufficient conditions for such reconstruction in terms of an adequate notion of density of the set of samples. The most classical density theorems, due to Shannon and Beurling, involve bandlimited functions (that is, functions whose Fourier transforms are supported on the unit interval) and provide a precise geometric characterization of all configurations of points that lead to reconstruction. I will present modern variants of these results and their applications in other fields of analysis.

Spectral geometry of CR manifolds

Speaker: Yunus Zeytuncu (University of Michigan Dearborn)

Date: November 23, 2020

Abstract: In this talk, we look at the spectrum of the Kohn Laplacian on spheres \mathbb{S}^{2n-1} and the Rossi sphere \mathbb{S}_t^3 . We relate spectral calculations to the geometry of the manifolds. In the first part, we use the lower bounds on the essential spectrum to understand the embeddability of the Rossi sphere. In the second part, we compute the leading coefficient in the asymptotic expansion of the counting function on \mathbb{S}^{2n-1} by a Tauberian argument, and we relate it to the volume of the spheres.

A unified method for commutators of Calderón-Zygmund operators

Speaker: Kabe Moen (University of Alabama)

Date: December 2, 2020

Abstract: We present a unified method to obtain weighted estimates of linear and multilinear commutators with BMO functions, that is amenable to a plethora of operators and functional settings. Our approach elaborates on a commonly used Cauchy integral trick, recovering many known results but yielding also numerous new ones. We also look at some new two weight bump conditions for commutators.