Southeastern Analysis Meeting 39: Titles and Abstracts

Plenary and Semi-Plenary Talks

Hardy spaces, BMO and related function spaces

Speaker: Galia Dafni - Concordia University

Abstract: Real Hardy spaces and BMO are well-known function spaces used in harmonic analysis and PDE, and have given rise to many related function spaces. This talk will focus on a few of these variants and review results on duality, boundedness of operators, and extension domains.

What can chicken McNuggets tell us about symmetric functions, positive polynomials, random norms, and AF algebras?

Speaker: Stephan Ramon Garcia - Pamona College

Abstract: Numerical semigroups are combinatorial objects that lead to deep and subtle questions. With tools from complex, harmonic, and functional analysis, probability theory, algebraic combinatorics, and computer-aided design, we answer virtually all asymptotic questions about factorization lengths in numerical semigroups. Our results yield uncannily accurate predictions, along with unexpected results about symmetric functions, trace polynomials, and the statistical properties of certain AF C*-algebras.

Partially supported by NSF Grants DMS-1800123 and DMS-2054002. Joint work (in various combinations) with K. Aguilar, A. Böttcher, Á. Chávez, L. Fukshansky, M. Omar, C. O'Neill, J. Volčič and undergraduate students J. Hurley, G. Udell, T. Wesley, S. Yih.

Sobolev Regularity of Singular Integral Operators on Domains

Speaker: Walton Green - Washington University in St. Louis

Abstract: Extending the Sobolev theory of quasiconformal and quasiregular maps in the complex plane to subdomains motivates our investigation of Sobolev regularity of singular integral operators on domains. We introduce new paraproducts which lead to higher order $T1$-type testing conditions. Furthermore, we have weighted Sobolev estimates for the compressed Beurling transform which imply self-improving Sobolev regularity for certain quasiregular distributions. This is joint work in progress with Francesco Di Plinio and Brett D. Wick.

A survey of some recent developments in free analysis

Speaker: Michael Jury - University of Florida

Abstract: This talk will survey some recent developments in "free analysis," in particular the interplay between multivariable operator theory and the rapidly growing area of noncommutative function theory, which at the most basic level provides a kind of functional calculus for noncommuting operators. The emphasis will be on the free analogue of the classical Hardy space and analogs of some of the familiar objects there (multipliers, inner-outer factorization, Toeplitz and Hankel operators, etc.) I will attempt a quick overview of

the development of the subject and the current state of affairs, highlighting applications to problems in more classical settings (such as the Dirichlet space and Drury-Arveson space), and some open problems.

Laplace eigenfunctions and the frequency function method

Speaker: Eugenia Malinnikova - Stanford University

Abstract: A classical idea in the study of eigenfunctions of the Laplace-Beltrami operator is that they behave like polynomials of degree corresponding to the eigenvalue. We will discuss several properties of eigenfunctions which confirm this idea, including the Bernstein and Remez inequalities. As a corollary, we will formulate a local version of the celebrated Courant theorem on the number of nodal domains of eigenfunctions. One of the main tools in our arguments in the frequency function. We will show some applications of the frequency function on the boundary of not very smooth domains to Dirichlet Laplace eigenfunctions.

Persistence of superoscillations

Speaker: Elodie Pozzi - St. Louis University

Abstract: Superoscillation is a phenomenon in which a signal which is globally band-limited can contain local segments that oscillate faster than its fastest Fourier components. This phenomenon plays an important role in quantum physics and engineering. It is initially attributed to Y. Aharonov and similar concepts were known to I. Daubechies. A function showing superoscillations is called a superoscillating function. The study of superoscillating functions is also interesting from the mathematical point of view. In this talk, we will present results on superoscillating functions and in particular, the persistence of superoscillations under the Schrödinger equation with different potentials. This talk is based on joint work with F. Colombo, I. Sabadini and B. D. Wick.

An Operator Theoretic Approach to System Identification

Speaker: Joel Rosenfeld - University of South Florida

Abstract: In this talk we discuss the connection between kernel spaces, system identification, and certain operators related to dynamical systems. This emphasizes our group's effort towards Dynamic Mode Decompositions and other basis function free approaches to system identification.

Almost sure local well-posedness of the nonlinear Schrödinger equation using directional norms.

Speaker: Gennady Uraltsev - University of Virginia

Abstract: Local well-posedness results for the nonlinear Schrödinger equation on the Euclidean space is well-undestood: it holds for initial data with regularity above the energy-critical threshold. In the probabilistic setting, local well-posedness holds for much rougher initial data. We show how directional maximal and local-smoothing estimates can be used to recover deterministic results and obtain probabilistic results in less regular spaces. In particular, when $d = 3$ we show that local-wellposedness holds for almost all initial data in H^S with $S > 0$.

Compactness of the Bloom sparse operators and applications

Speaker: Naga Manasa Vempati - Georgia Institute of Technology

Abstract: We discuss the characterization of compactness for the sparse operator (associated with symbol in weighted VMO space) in the two-weight setting on the spaces of homogeneous type. As a direct application we obtain the compactness characterization for the maximal commutators with respect to the weighted VMO functions and the commutator of Calderón–Zygmund operators on the homogeneous spaces. We will look at the applications of this approach to multilinear Bloom setting.

Contributed Talks

Umbrella Theorems in a Reproducing Kernel Hilbert Space

Speaker: Travis Alvarez - Clemson University

Abstract: Umbrella Theorems have their roots in uncertainty principles that arise from the relationship between functions and their Fourier transforms. However, these principles can also be found in reproducing kernel Hilbert spaces. In this talk we will explore some of the umbrella theorems and how they can be generalized to reproducing kernel Hilbert spaces.

An Optimal Approximation Problem for Noncommutative Polynomials

Speaker: Palak Arora - University of Florida

Abstract: Motivated by recent work on optimal approximation by polynomials in the unit disk, we consider the following noncommutative approximation problem: for a polynomial f in d noncommuting arguments, find an nc polynomial p_n, of degree at most n, to minimize

$$c_n := \|p_n f - 1\|^2.$$

(Here the norm is the ℓ^2 norm on coefficients.) We show that $c_n \to 0$ if and only if f is nonsingular in a certain nc domain (the *row ball*). As an application we give a new, elementary, proof of a theorem of Jury, Martin, and Shamovich on cyclic vectors for the d-shift. (This is joint work with M. Augat, M. Jury, and M. Sargent.)

What relations can two noncommutative functions satisfy?

Speaker: Meric Augat - University of South Florida

Abstract: A remarkable result in noncommutative algebra says that given any two noncommutative rational functions, either they commute, or they satisfy no non-trivial relations. Inspired by this result, we investigate an analogous conjecture for noncommutative analytic functions. Using the Agler-McCarthy Implicit Function Theorem, we are led to several conjectures about the nature of noncommutative analytic functions.

Sliced optimal partial transport

Speaker: Yikun Bai - Vanderbilt University

Abstract: Optimal transport (OT) has become exceedingly popular in machine learning, data science, and computer vision. Optimal Partial Transport (OPT) is a recently proposed solution to this limitation. Similar to the OT problem, the computation of OPT relies on solving a linear programming problem, which can become computationally prohibitive. In this paper, we propose an efficient algorithm for calculating the OPT problem between two non-negative measures in one dimension. Next, following the idea of sliced OT distances, we utilize slicing to define the sliced OPT distance. Finally, we demonstrate the computational and accuracy benefits of the sliced OPT-based method in various numerical experiments.

On Weyl-Titchmarsh functions and Schrödinger L-systems

Speaker: Sergey Belyi - Troy University

Abstract: The focus of this talk is set on classical objects in Schrödinger operators theory, the original Weyl-Titchmarsh functions and their connections with conservative L-systems with Schrödinger operators. We study realizations generated by the original Weyl-Titchmarsh functions $m_\infty(z)$ and $m_\alpha(z)$. It is shown that the Herglotz-Nevanlinna functions $(-m_\infty(z))$ and $(1/m_\infty(z))$ can be realized as the impedance functions of the corresponding Schrödinger L-systems sharing the same main dissipative operator. These L-systems are presented explicitly and related to Dirichlet and Neumann boundary problems. Similar results but related to the mixed boundary problems are derived for the Herglotz-Nevanlinna functions $(-m_\alpha(z))$ and $(1/m_\alpha(z))$. The talk is based on joint work with E. Tsekanovskii.

On a Szegő-Type Limit Theorem for Berezin-Toeplitz Operators

Speaker: Trevor Camper - Clemson University

Abstract: Szegő Limit Theorems have been used to obtain semi-classical Weyl Laws for Berezin-Toeplitz Operators. In this talk, a new Szegő-Type Limit Theorem for a semi-classical setting will be presented. From this, semi-classical Weyl Laws for Berezin-Toeplitz Operators will be presented for a variety of function spaces. Finally, some applications to Gabor and Wavelet systems will be presented.

Variant of Paulsen Problem in probabilistic frames and optimal transport

Speaker: Dongwei Chen - Clemson University

Abstract: The Paulsen problem is a basic problem in frame theory saying that every ϵ-nearly equal norm Parseval frame in d dimensions is within squared distance $O(\epsilon d^{13/2})$ of an equal norm Parseval frame. A variant of Paulsen Problem is that the closest Parseval frame to a given frame is the canonical dual Parseval frame. In this talk, we will talk aobut a similar variant of Paulsen Problem for probabilistic frame, which is a probability measure on $\mathbb{R}^d$ with an invertible second-moment matrix. We show that there exists a unique closest tight probabilistic frame with unit norm to a given probabilistic frame, where the distance is quantified by the 2-Wasserstein metric in optimal transport.

Prime ends and weak * generators

Speaker: Joseph Cima - University of North Carolina, Chapel Hill

Abstract: We have examples of the types of prime ends in the theory of univalent mappings and how they relate to weak * generators of H^∞.

Bounded Point Derivations and Functions of Bounded Mean Oscillation

Speaker: Stephen Deterding - Marshall University

Abstract: A bounded point derivations is a special kind of linear functional that is widely studied in the theory of rational approximation on the complex plane. This is because the existence of a bounded point derivation at a boundary point implies that for every convergent sequence of approximating functions, the sequence of derivatives also converges at that point, which does not happen in general. We will discuss how to determine when bounded point derivations exist for a variety of different types of approximations, but with a focus on approximations by functions of bounded mean oscillation, for which we have proven necessary and sufficient conditions.

Algebra generated by Toeplitz operators on the Fock space

Speaker: Vishwa Dewage - Clemson University

Abstract: We study the full Toeplitz algebra via convolutions of operators and the lapacian of the Berezin transform. We present a new class of operators that are dense in the Toeplitz algebra.

It is known that algebra generated by radial Toeplitz operators is commutative and is isomorphic to the space of bounded sequences that are uniformly continuous with respect to the square-root metric. We present a new proof of this.

This is a joint work with Dr. Mishko Mitkovski.

A new weighted spectral geometric mean

Speaker: Trung Hoa Dinh - Troy University

Abstract: In this talk we introduce a new weighted spectral geometric mean. We study its properties and some related inequalities. We also discuss the Lie-Trotter formula for the new object.

Using Shoenberg's Theorem to Expand on Trade Models

Speaker: Austin Jacobs - University of Florida

Abstract: In this talk we will discuss an application of Shoengberg's theorem in extending a trade model of Mossay and Tabuchi to larger network sizes. This reframing provides a simplification of a proof used in the original three country model, immediately extending it to four countries and allows for additional framework to investigation of pathological network types.

Invariant subspaces of composition operators on S^2

Speaker: Bhupendra Paudyal - Central State University

Abstract: Suppose p is an analytic self map of the complex unit disk and S^2 is the space of the holomorphic functions such that their first derivative belong the Hardy space. The composition operator C_p and the shift operator M_z on S^2 are defined by $C_p(f) = f \circ C_p$. In this work, the invariant subspaces of composition operators on S^2 are studied. Additionally, the conditions for some shift-invariant subspaces to be invariant for composition operators are discussed.

On distributions associated with tossing unfair coins and rolling unfair dice infinitely often

Speaker: Douglas Pfeffer - University of Tampa

Abstract: Flipping an unfair coin infinitely often yields a sequence of 0s and 1s. If we form $X = \sum_{i=1}^{\infty} \frac{x_i}{2^i}$, where $x_i \in \{0, 1\}$, then we associate this process with the binary representation of some $X \in [0, 1]$. We ask: Given $y \in [0, 1]$, what is $\mathbb{P}[X \leq y]$? What if we instead roll an unfair n-sided die? In part, we develop a recursive definition for these CDFs and show that it is both singular and fractal-like. This work stems from a collaboration with J. Darby Smith and William Severa at Sandia National Labs on Probabilistic Neural Computing.

Self-Maps of the Unit Ball in Several Complex Variables

Speaker: Michael Pilla - Ball State University

Abstract: In this talk, we investigate the conditions under which a class of maps in $\mathbb{C}^N$, known as linear fractional maps, are self-maps of the unit ball. In particular, we take a new approach to the problem, improving a result by Richman (2002) and discovering new properties of these extraordinary maps along the way. This is part of a larger joint project with Brittney Miller and Chris Felder.

Compactness of Toeplitz operators with continuous symbols on pseudoconvex domains in $\mathbb{C}^n$

Speaker: Tomas Miguel Rodriguez - University of Toledo

Abstract: Let Ω be a bounded pseudoconvex domain in $\mathbb{C}^n$ with Lipschitz boundary and ϕ be a continuous function on $\overline{\Omega}$. We show that the Toeplitz operator T_ϕ with symbol ϕ is compact on the weighted Bergman space if and only if ϕ vanishes on the boundary of Ω. We also show that compactness of the Toeplitz operator $T_\phi^{p,q}$ on $\overline{\partial}$-closed (p,q)-forms for $0 \leq p \leq n$ and $q \geq 1$ is equivalent to $\phi = 0$ on Ω.

Recover All Coefficients in Second-Order Hyperbolic Equations from Finite Sets of Boundary Measurements

Speaker: Scott Scruggs - Clemson University

Abstract: We consider the inverse hyperbolic problem of recovering all spatial dependent coefficients, which are the wave speed, the damping coefficient, the potential coefficient, and the gradient coefficient, in a second-order hyperbolic equation defined on an open bounded domain with a smooth enough boundary. We show that by appropriately selecting finite pairs of initial conditions, we can uniquely and Lipschitz stably recover all those coefficients from the corresponding boundary measurements of their solutions. The proofs are based on sharp Carleman estimate, continuous observability inequality and regularity theory for general second-order hyperbolic equations. We will end this talk by discussing future projects. This is joint work with Dr. Shitao Liu and Antonio Pierrottet.

Applied analysis for better AI methods

Speaker: Himanshu Singh - University of South Florida

Abstract: Traditional topics of complex analysis has been proved to be the boon for the development of learning algorithms for the Artificial Intelligence (AI) methods. One such complex analysis function is $\varphi_\epsilon(|z|) = \exp(-\epsilon|z|^2)$ for $z \in \mathbb{C}$ which is native to the area measure of Bergman-Segal-Fock space. The aforementioned function has been pivotal for the mathematicians, primarily because of its direct applications to the Fourier analysis, reproducing kernel theory and coherent states in quantum physics.

Let x and y in $\mathbb{R}^{n<\infty}$ and let the squared Euclidean metric be $\mathfrak{m}(x,y) = \|x-y\|_2^2$. Then, the celebrated Gaussian radial basis function is explicitly given as $K_G(x,y) = \varphi_\epsilon(r) \circ \mathfrak{m}(x,y)$. Gaussian radial basis function has yielded better and optimal results for various important AI methods. This presentation aims to introduce a *better* kernel function which outperforms the traditional Gaussian radial basis function on various venues of learning algorithms important to ML and AI methods. After settling with the key analysis of the newly introduced kernel function, empirical results will also be presented and discussed. These empirical results

circulates in the themes of *Kernel Regression*, *Support Vector Machine* and *Neural Network* applications.

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Hardy-Littlewood and Paley inequalities for $H^{\log}$ and related spaces

Speaker: Alan Sola - Stockholm University

Abstract: Reporting on joint work with O. Bakas, S. Pott, and S. Rodriguez-Lopez, I will present some generalizations of classical inequalities of Hardy-Littlewood, Paley, Duren-Shields, and others to the setting of $H^{\log}$ and other Hardy-Orlicz spaces.

Multiplier weak type inequalities for maximal operators and singular integrals

Speaker: Brandon Sweeting - University of Alabama

Abstract: We discuss a kind of weak type inequality for the Hardy-Littlewood maximal operator and Calderón-Zygmund singular integral operators that was first studied by Muckenhoupt and Wheeden and later by Sawyer. This formulation treats the weight for the image space as a multiplier, rather than a measure, leading to fundamentally different behavior. In this talk, I will discuss quantitative estimates obtained for A_p weights, $p > 1$, that generalize those results obtained by Cruz-Uribe, Isralowitz, Moen, Pott and Rivera-Ríos for $p = 1$. I will also discuss an endpoint result for the Riesz potentials.

Interpolating sequences for pairs of spaces

Speaker: Georgios Tsikalas - Washington University in St. Louis

Abstract: We characterize interpolating sequences for pairs of Hilbert function spaces $(\mathcal{H}_s, \mathcal{H}_\ell)$, where s, ℓ are reproducing kernels and s is a complete Pick factor of ℓ. Specifically, we prove that a sequence is interpolating if and only if it generates a Carleson measure for $\mathcal{H}_s$ and is n-weakly separated by ℓ for every $n \geq 2$, the latter condition lying in-between weak separation and strong separation by ℓ. We also exhibit examples to show that n-weak separation cannot, in general, be replaced by weak separation by ℓ, thus answering a question of Aleman, Hartz, McCarthy and Richter.

Optimal Transport Approach to Michael-Simon Inequalities

Speaker: Kai-Hsiang Wang - Northwestern University

Abstract: I will talk about a generalization of McCann's theorem in optimal transport theory to a submanifold setting and use it to prove Michael-Simon inequalities.

Sharp Hölder Regularity for Nirenberg's Complex Frobenius Theorem

Speaker: Liding Yao - The Ohio State University

Abstract: Nirenberg's famous complex Frobenius theorem gives necessary and sufficient conditions on a locally integrable structure for when it is equal to the span of some real and complex coordinate vector fields. In the talk I will discuss some differential complexes and how some of the notions make sense in the non-smooth setting. For a $C^{k,s}$ complex Frobenius structure, we show that there is a $C^{k,s}$ coordinate chart such that the structure

is spanned by coordinate vector fields which are $C^{k,s-\varepsilon}$ for all $\varepsilon > 0$. Here the $\varepsilon > 0$ loss in the result is optimal.

Normalizable sequences

Speaker: Pu-Ting Yu - Georgia Institute of Technology

Abstract: Let H be a Hilbert space. We study the sequences which can be frames or Bessel sequences or satisfy a lower frame condition after being normalized. We will also apply our results to partially answer some existing open problems.

Geometric Maximal Operators and BMO on product bases

Speaker: Hong Yue - Georgia College and State University

Abstract: This is a joint work with Galia Dafni and Ryan Gibara. We consider the problem of the boundedness of maximal operators on BMO on shapes in $\mathbb{R}^n$. We prove that for bases of shapes with an engulfing property, the corresponding maximal function is bounded from BMO to BLO, generalising a known result of Bennett for the basis of cubes. When the basis of shapes does not possess an engulfing property but exhibits a product structure with respect to lower-dimensional shapes coming from bases that do possess an engulfing property, we show that the corresponding maximal function is bounded from BMO to a space we define and call rectangular BLO.